THE EFFECTS OF LARGE DISPLACEMENTS ON THE EARTHQUAKE RESPONSE OF TALL CONCRETE FRAME STRUCTURES

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Summary:

Many analytical techniques have been developed for the inelastic analysis of multi-storey framed structures subjected to earthquake excitations. Most of these analyses have ignored the influence of the geometric effects due to the large lateral displacements on the response of the structure. However, there has been an increasing interest recently in these second order effects and, in particular, their consequences for the design of the column members in the frame.

This paper describes the modification of an inelastic frame analysis to include the effects of large displacements and then the application to three typical New Zealand concrete structures subjected to a variety of earthquake excitations. Comparisons are then made with results obtained from analyses ignoring these effects. The results are then reviewed in order to determine the nature of the problems, to determine when these second order effects should be considered and discusses methods of limiting these displacements.

INTRODUCTION:

Most analyses of multi-storeyed reinforced concrete frames subjected to earthquake excitation have ignored the secondary effect due to the combination of gravitational forces and large displacements. This is often referred to as the P-Delta effect. For frames where the lateral displacements are relatively small and the gravitational forces are not great, i.e. for moderately low structures subject to small to moderate earthquake shaking, the P-Delta effect is not great and can justifiably be neglected. However, with the taller structures now being built in seismic areas there has developed an increasing interest in determining the magnitude of these P-Delta effects and on methods to reduce the influence of the P-Delta effect by controlling the resulting lateral displacements (1, 2, 3).

For reinforced concrete frames designed for seismic conditions in New Zealand, adequate ductility must be designed into the column and beam members - as well as the joints - with there being a limit to the maximum achievable ductility. Often inelastic analyses are carried out in order to check on the maximum ductilities required and maximum deflections, plastic rotations, etc, when the frame is subjected numerically to the effects of a particular earthquake record.

The problems to resolve are (i) at what level of lateral displacement are P-Delta effects considered to be significant when compared to the initial analyses carried out ignoring these effects, and (ii) if they are significant, should control of the deformation be best achieved by increasing the stiffness of the members or by increasing the strength of the members[2,3].

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METHOD OF ANALYSIS

The computer analysis used is a development of the dynamic time-history analysis program for inelastic framed structures originally developed by Sharpe[4]. The original program has now been extended and modified so that the ultimate strengths of the members are given in terms of maximum positive and negative moments for beams, while for column members sufficient information can be given to define the maximum extent of the load-moment interaction curve in tension and compression. Different damping models are possible based on either the initial or the tangent stiffness.

The program has been further modified to include the effects of (large) displacements that occur during the analysis so that at every time step the member properties are redefined in terms of the updated coordinates of all joints in the frame. The disadvantage of such an analysis is however a very great increase in the computational effort required to effect the complete analysis since changes in both axial forces and geometry must be monitored.

For a rectangular frame such as that occurring in many multi-storey reinforced concrete buildings, the reduction in lateral stiffness due to an increase in the column axial forces on one side of the structure will largely be offset by the increase in lateral stiffness due to the corresponding decrease in axial forces in the columns on the opposite side. Therefore provided vertical accelerations are negligible, the total change in lateral stiffness due to the combination of the interstorey drifts and the column axial forces during the seismic analysis may be considered to be constant and proportional only to the gravitational forces on the columns. This suggests an approximation to the P-Delta effect whereby it is calculated using the initial gravitational loads in the columns and the instantaneous drifts rather than using the instantaneous values of the column loads at each time-step in the time-history analysis. Thus the reduction of lateral stiffness due to the gravitational loads could be carried out at the beginning of the analysis resulting in negligible increase in the computational effort when compared with analyses that ignore the P-Delta effect. The results described in this paper use the latter simplification for their analyses.

FRAMES ANALYSED:

These were reinforced concrete frames designed in accordance with proposed revisions to the New Zealand concrete design code [5] and loaded to the provisions of New Zealand Standard 4203[6] for seismic Zone A. This latter standard in its earthquake provisions provides for equivalent static force analyses to be carried out to determine the likely seismic resistance of the frame. The equivalent horizontal load to be applied depends on several multiplicative factors, the pricinpal one of which is the base shear coefficient which, for buildings in seismic Zone A, is 0.15 for fundamental periods below 0.45 secs and 0.075 for periods above 1.2 secs with a linear variation between these two periods. Other factors relate to the importance of the building (I = 1.0 to 1.6), the structural type (S = 0.8 to 2.5) the structural material (M = 0.8 to 1.2) and the risk (R = 1.0 to 1.1).

The frames were typical two bay interior frames for buildings of 6, 12 and 18 floors. No torsional effects for the building as a whole were taken into account and the frames were taken as being loaded in their own plane. The building dimensions and member sizes used for each frame are shown in Figure 1 - 3 and Tables 1 - 3. Further details and discussion of the frame design are given by Jury[17].

To simulate an equivalent design for seismic Zone C (having the lowest base shear coefficient), the beam and column strengths were reduced by one third.

EARTHQUAKE RECORDS USED:

The records that were used for various analyses were those recorded for El Centro (May 1940, N-S), Parkfield, Bucharest, Pacoima and the Artificial A2.

The Pacoima record was used since it represents the maximum likely intensity earthquake that would be expected to occur in that part of New Zealand that lies in seismic Zone A. The El Centro record was used since the code level seismic shears and design drift limits are based on the likely response to an El Centro type earthquake, with the code assuming a ductility level of four and the actual likely earthquake drifts being 2.4 times larger than those computed using the equivalent static lateral forces from the code.

The Parkfield earthquake record was chosen as being typical of that which might be experienced at a site close to the epicentre of a medium intensity earthquake and situated within the lobe of maximum energy release. The Bucharest record is typical of that likely to be experienced at a site some distance from the epicentre of a large intensity earthquake but situated within the lobe of maximum energy release.

The Artificial A2 record was used with the 18 storey frame only, as it was likely to excite the frame more severely than the Pacoima record because of the longer natural period of this frame.

ANALYSES CARRIED OUT:

A Rayleigh damping model[4] using the initial stiffness was assumed with the damping taken to be 8% of critical on two specified modes. These modes were:

6 storey structure - modes 1 and 6

12 storey structure - modes 1 and 10

18 storey structure - modes 1 and 15

As a check, damping levels for the other modes were automatically calculated by the program using the standard Rayleigh damping formula. Using 8% critical damping on the above modes meant that for the analyses all modes were subcritically damped.

6 Storey Frame -

This was analysed elastically and inelastically under the El Centro and Pacoima reocrds, details being as follows:

(a) elastic (i) standard frame;

(ii) the above with P-Δ secondary effects included;

(b) inelastic (i) standard frame;

(ii) the above but with P-Δ effects included.

12 Storey Frame -

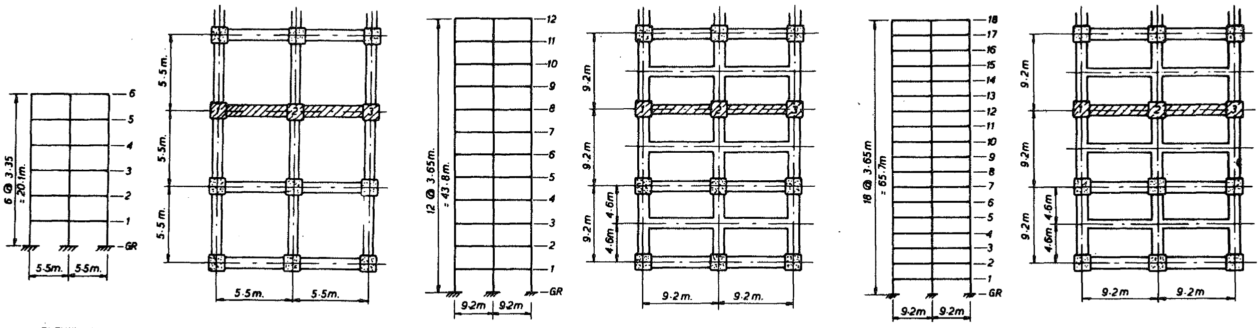
The majority of analyses were carried out on this frame and these were:

(a) elastic (i) standard frame subjected to code loading;

(ii) standard frame subjected to code loading and allowing for P-Δ effects;

(iii) standard frame analysis, and

(iv) the above with P-Δ secondary effects included.



Figs. 1, 2 and 3 – Building dimensions for 6, 12 and 18 storey buildings.

	FLOOR	
	1 - 3	4 - 6
Main Beams	600 x 350	580 x 350
Cols. 1 & 3	500 x 450	450 x 450
Col. 2	550 x 550	500 x 500

Note: (i) Slab thickness is 120 mm throughout

(ii) f'_c = 28 MPa throughout

Table 1 – Member Dimensions for Six-Storey Frames

	FLOOR		
	1 - 6	7 - 8	9 - 12
Main Beams	900 x 400	850 x 400	800 x 400
Secondary Beams	750 x 400	750 x 400	750 x 400
Col. 1 & 3	700 x 500	650 x 500	600 x 500
Col. 2	725 x 725	675 x 675	625 x 625

Note: (i) Slab thickness is 160mm throughout

(ii) f'_c = 28 MPa throughout

Table 2 – Member Dimensions for Twelve Storey Frame

	FLOOR				
	1 - 6	7 - 9	10 - 12	13 - 15	16 - 18
Main Beams	1000 x 550	950 x 550	900 x 500	850 x 450	800 x 400
Secondary Beams	750 x 450	750 x 450	750 x 450	750 x 450	750 x 450
Cols. 1 & 3	1000 x 650	800 x 650	750 x 650	700 x 650	650 x 650
Col. 2	1000 x 1000	1000 x 1000	900 x 900	800 x 800	700 x 700

NOTE: (i) Slab thickness is 160 mm throughout

(ii) f'_c for floors 1 to 6 = 35 MPa

7 to 18 = 28 MPa

Table 3 – Member Dimensions for Eighteen Storey Frame

(b) inelastic (i) standard frame;

(dynamic) (ii) the above with P-Δ effects included.

Two further analyses were also carried out to simulate the behaviour of frames designed for seismic action in Zone C, these were:

(c) inelastic (i) beam and column strengths both reduced by one third, and

(ii) the above with P-Δ effects included.

18 Storey Frame

Analyses carried out using El Centro and the Artificial A2 records were:

(a) elastic (i) standard analysis;

(ii) the above with P-Δ effects included.

(b) inelastic (i) standard analysis, and

(ii) the above with P-Δ effects included.

RESULTS OF THE ANALYSES:

6 Storey Frame

The horizontal displacement envelopes are illustrated in Figure 4 while the interstorey drifts are shown in Figure 5. The maximum plastic hinge rotations in the beams and the ground floor columns are shown in Figure 6.

The inclusion of P-Delta effects in the analysis using the Pacoima Dam record increases the deflections slightly, though the drifts are actually smaller over the top half of the building than for the standard analysis.

The top storey deflections and interstorey drifts under Parkfield and Pacoima are greater than the code allowable values though the maximum plastic hinge rotations are capable of being achieved in well detailed structures.

The displacement response of this frame to the Pacoima Dam excitation is shown in Figure 7. This shows that the maximum deflections occur just after 3 seconds and that after 8 seconds of the excitation there is a mean permanent top storey deflection of approximately 100 mm.

From Figures 4 to 6 it is apparent that for this 6 storey frame the moderate lateral drifts together with the low column axial forces result in a negligible change due to the inclusion of the P-Delta effect.

12 Storey Frame

The horizontal displacement envelopes from the Zone A frame analyses are shown in Figures 8 and 11 while those for the Zone C frame are shown in Figure 19.

The maximum interstorey drifts are illustrated in Figures 9, 12 and 20, and the plastic hinge rotations in the beams and the ground floor columns are shown in Figures 10, 13 and 21.

(a) El Centro Earthquake Analysis

For the elastic analysis based on the code distribution of lateral load it can be seen from Figure 8 that the inclusion of secondary effects caused by the lateral displacement at each floor causes a small increase in the deflections. Because the code distribution of the lateral load is derived from an El Centro response, it is not surprising that if these deflections are increased by a factor of 2.4, then they are close to those of the envelope for the dynamic analysis. However it can be seen that the inclusion of P-Delta effects into the dynamic analysis of the frame actually produces a reduction in this case, in the deflections at all levels in the structure.

One possible reason for the reduction of the response under P-Delta is the lengthening of the natural periods caused by the inclusion of the gravitational axial forces into the analysis. This appears to reduce the response to a greater extent than the P-Delta effect increases it.

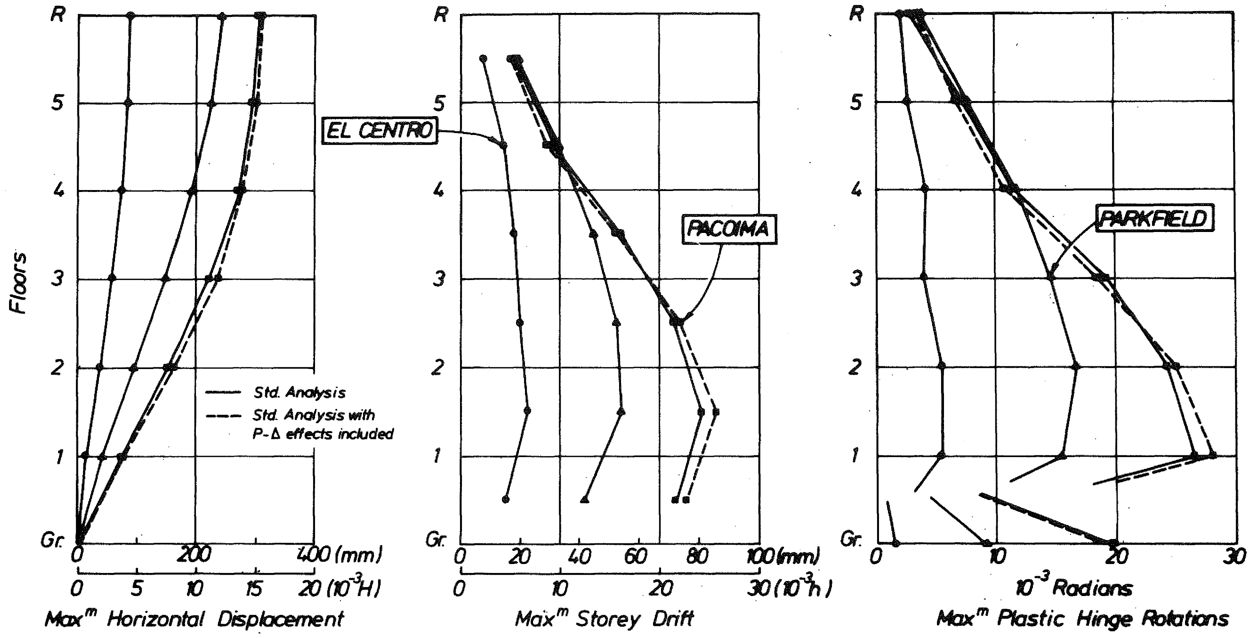
Figure 9 compares the maximum interstorey drifts obtained from each of the analyses. For the code loading analysis with P-Delta effects included, the drifts are uniform over most of the building height but reduce towards the top. In the case of the dynamic inelastic analyses there is some variation in the drifts at different floors up the building, particularly in the case of the standard analysis with P-Delta effects included. In no case does the drift under the El Centro earthquake record exceed the code drift limit of 0.01 times the storey height.

The maximum plastic hinge rotations in the beams at each floor level can be seen from Figure 10 to be reasonably uniform up the building and well below the limit at which it becomes impractical to detail the structure to have the necessary ductility.

(b) Pacoima Earthquake Analysis

The inclusion of P-Delta effects into the analysis of the standard frame increases the deflections considerably with a 40% increase in the top storey deflection, Figure 14 and 15 (compared with a reduction of 20% under El Centro).

Interstorey drifts are shown in Figure 12, 16 and 17, where it can be seen that the drifts are greatest between the first and second floors and then reduce fairly quickly to just above the mid-height of the building, reducing less rapidly from their to the top of the building. The maximum drifts are considerably greater than the allowable code maximum but decrease to below this value at the top of the building.



Figs. 4, 5 and 6 – Deflections, drifts and plastic rotations for 6 storey frame.

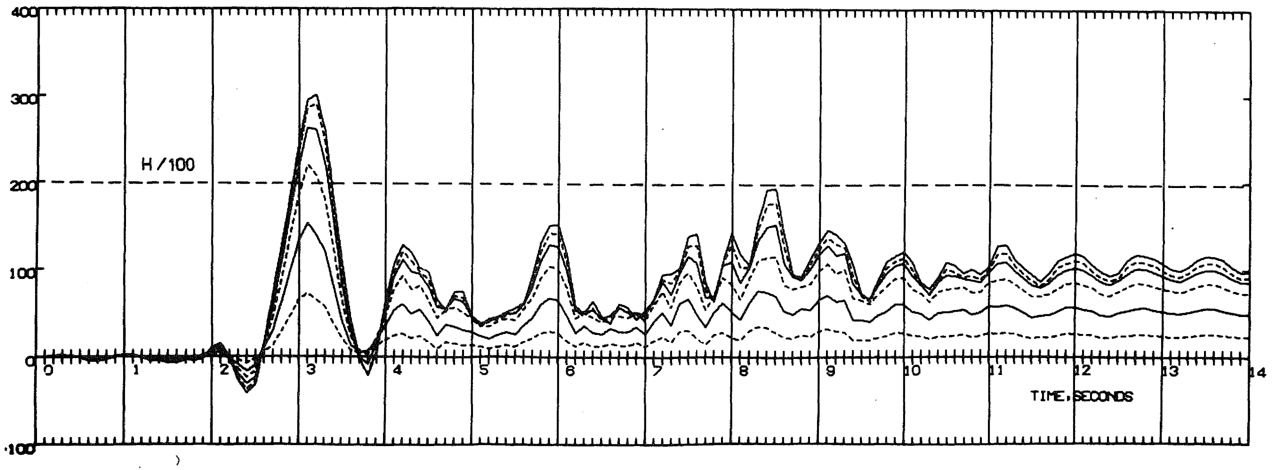
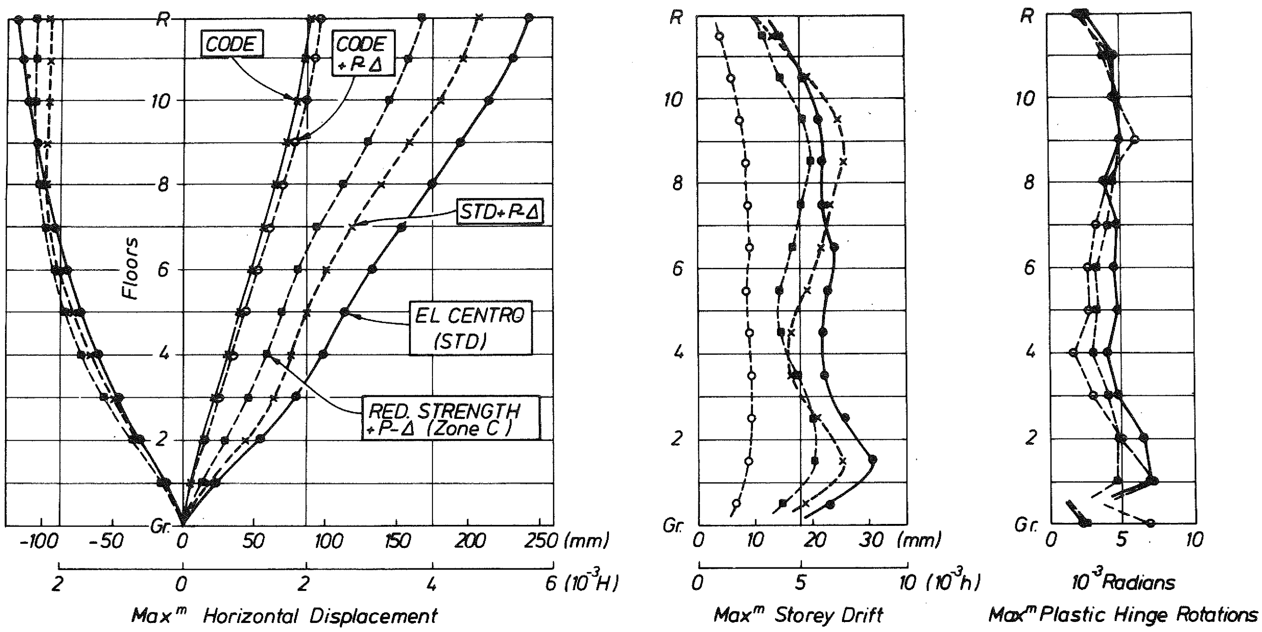
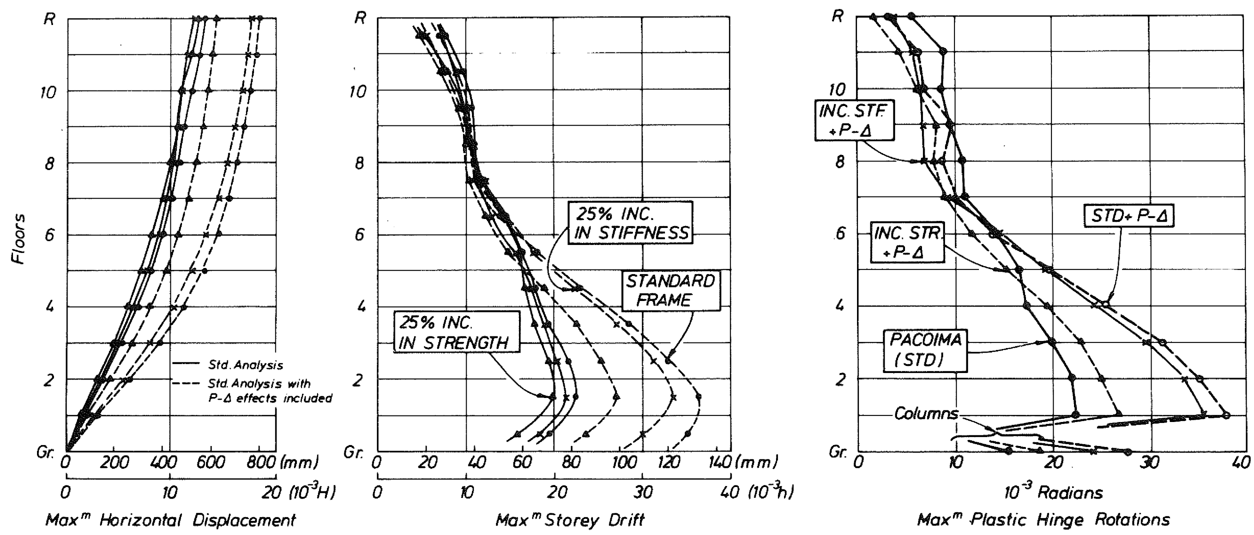


Fig 7. – 6 storey frame: displacement history for all floors, Pacoima earthquake.



Figs. 8, 9 and 10 — Deflections, drifts and plastic rotations for 12 storey frame under code loading and ElCentro Earthquake.



Figs. 11, 12 and 13 — Deflections, drift and plastic rotations for 12 storey frame under Pacoima earthquake

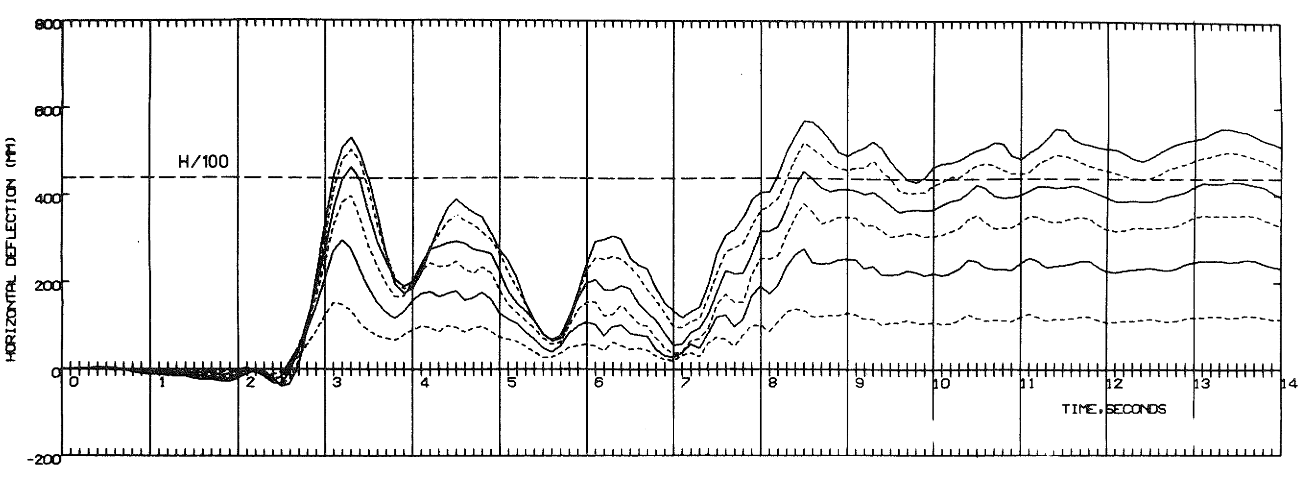


Fig. 14 — 12 storey frame: displacement history for even floors, Pacoima earthquake.

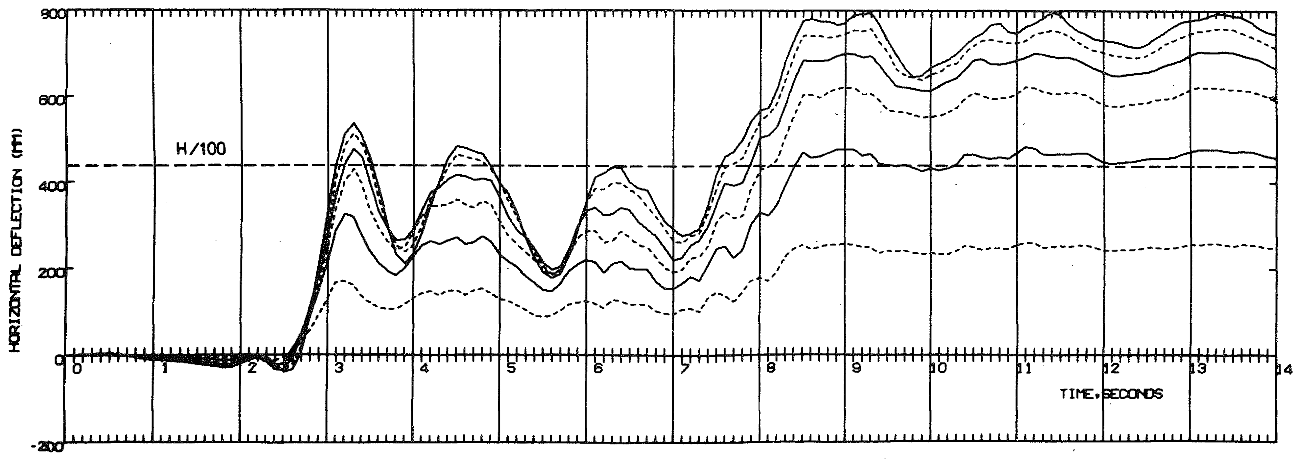


Fig. 15 — 12 storey frame with P—Delta: displacement history for even floors, Pacoima earthquake.

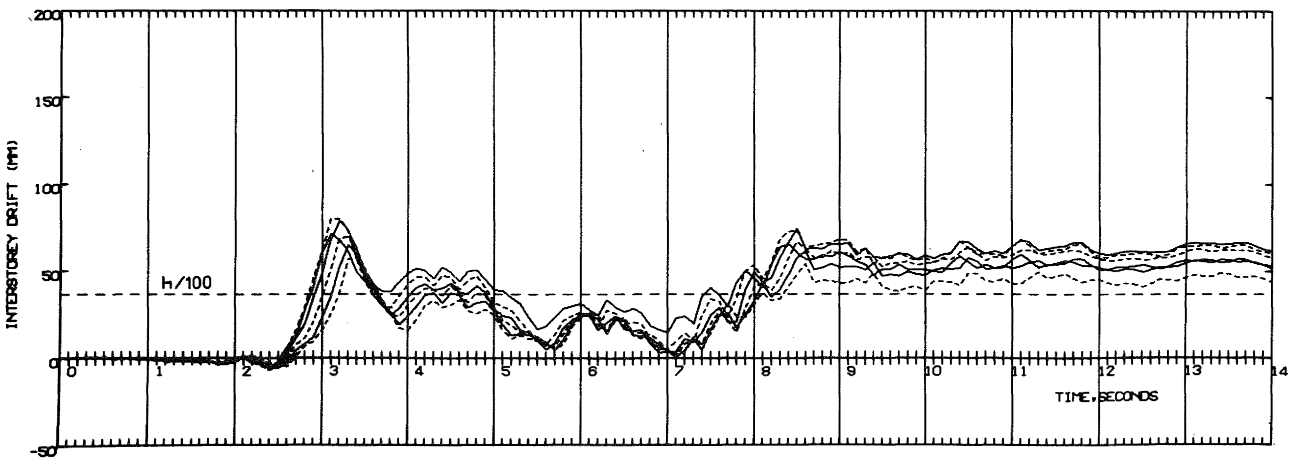


Fig. 16 — 12 storey frame: drift history for even floors, Pacoima earthquake.

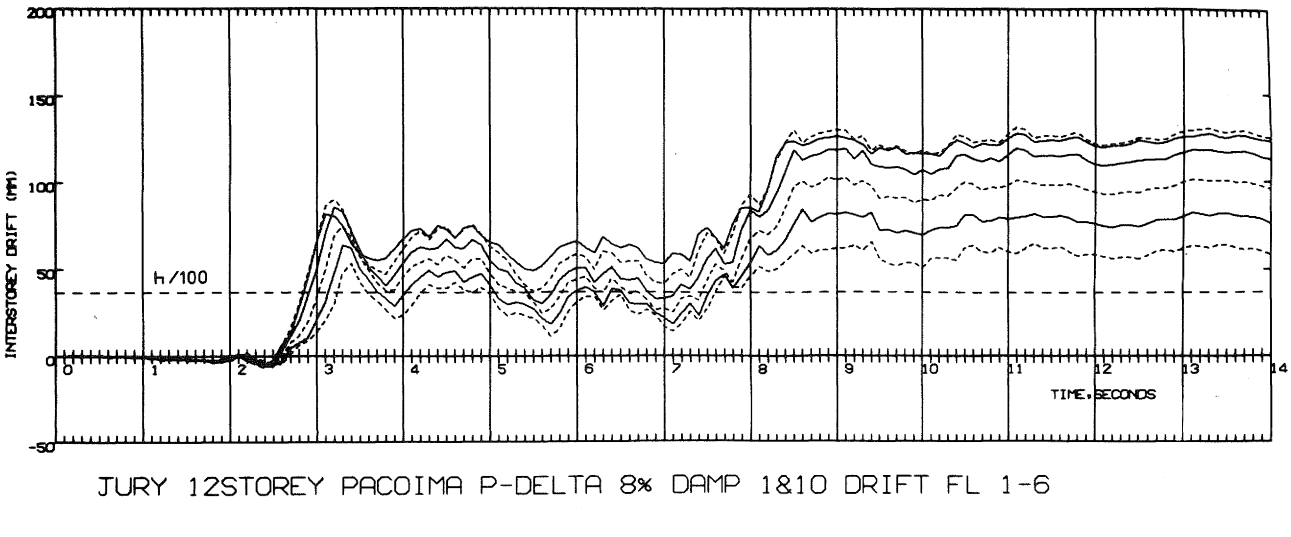


Fig. 17 — 12 storey frame with P—Delta: drift history for even floors, Pacoima earthquake.

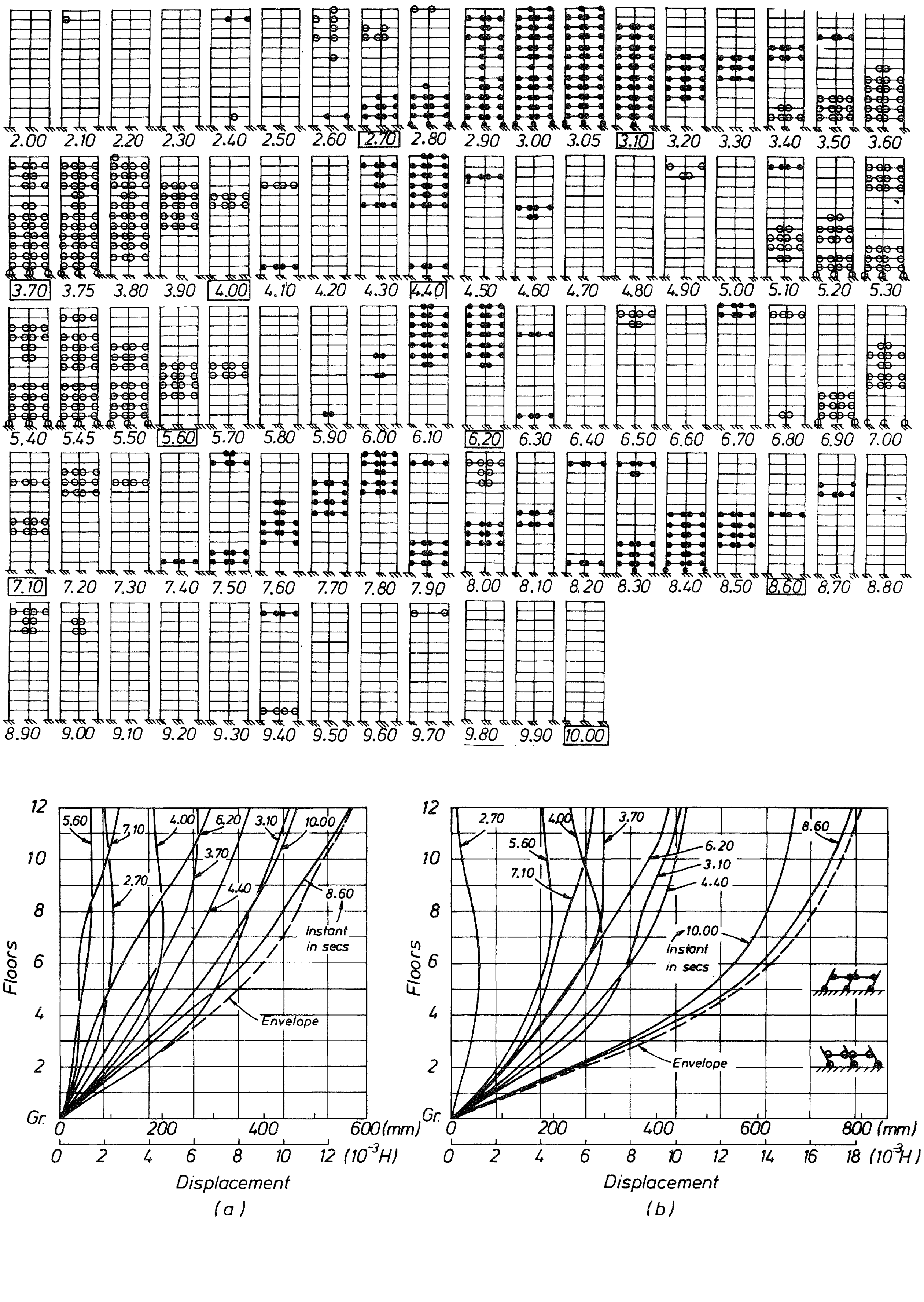


Fig. 18 – 12 storey frame: hinge patterns and deflected profiles at selected times (a) without P–Delta (b) with P–Delta.

In an endeavour to determine the most practical way of controlling the increase of the drifts due to the P-Delta effect, two further analyses were carried out. The first, increasing the stiffness of the structure by 25% but maintaining the original strength, showed an increase in maximum interstorey drift of 49% compared with an increase of 62% for the original frame. For the second analysis, where the strength was increased by 25% while maintaining the original stiffness, the increase in maximum interstorey drift was reduced to 21% of the non P-Delta value for the original frame. On the other hand, if P-Delta drifts are expressed in terms of the non P-Delta drift, then for the original frame we have an increase in drift of 62%. Increasing the frame stiffness as above gives a value of 56% while on increasing the strength the ratio falls to 37%. This indicates that if the behaviour of a frame influenced by P-Delta effects is to be improved by reducing the drifts, then increasing the strengths is much to be preferred over increasing the stiffnesses.

Figure 13 gives the beam plastic hinge rotations from the various analyses. Here it can be seen that for the analyses of the standard frame and for the frame with increased column stiffness, the rotations over the bottom three storeys exceed the value of 30×10^{-3} radians when P-Delta effects are included. The value of 30×10^{-3} radians for the plastic hinge rotation has been found experimentally to be the limiting value that can be achieved by means of careful detailing of the reinforcing. Maximum plastic rotations in the beams for the top half of the building are similar for all analyses and can be achieved without any great difficulty.

The development of hinge patterns during the excitation of the frame are illustrated in Figure 18, together with deflected shapes at selected time steps. Although the deflections are significantly different between the analyses with and without P-Delta effects, the hinge patterns are almost identical at each time step.

Zone C Frame Analyses

As mentioned previously for the dynamic analyses, cases C (i) and C (ii), the beam and column strengths were all reduced by one third below that for the standard frame in order to approximately model a frame with the lower level of seismic resistance permitted for Zone C.

Figure 19 shows the maximum horizontal displacement envelopes where it can be seen that the El Centro and Parkfield records produce top storey deflections that are less than 1% of the building height. On the other hand, the Bucharest and Pacoima records produce larger displacements where, in the case of the Pacoima earthquake, the top half of the building undergoes virtually the same maximum displacement.

Under the El Centro earthquake, the storey drifts are reasonably constant up the building, as can be seen in Figure 20. In the upper floors of the building the drifts are similar under all earthquake records, but in the lower three storeys the drifts for the Pacoima earthquake become very large and are many times greater than the code recommended limit of 0.01 times the storey height. For the Parkfield record, the maximum drifts in the bottom three storeys are approximately twice the code limit. The Bucharest record produces drifts greater than Parkfield and which, over the middle third of the building height, are similar to those produced by the Pacoima record.

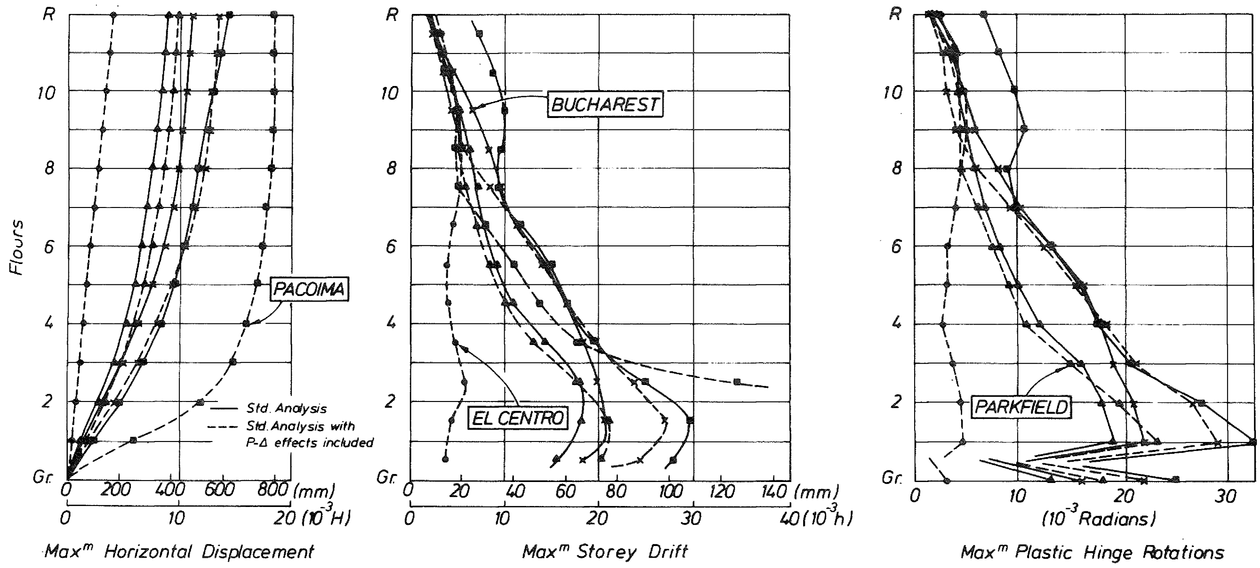
The maximum plastic rotations in Figure 21 show that for the El Centro record there is again not much variation up the height of the building but the maximum rotation occurs near the top of the building. For the other earthquakes, the maximum plastic rotation occurs in the first floor beams. In the case of the Pacoima record, the maximum plastic hinge rotation is in excess of the value that can be achieved even by the most careful detailing of the design. When P-Delta effects are included, it can be seen that the the Pacoima earthquake, the beams in the bottom storeys have plastic rotations greater than can be provided.

18 Storey Frame

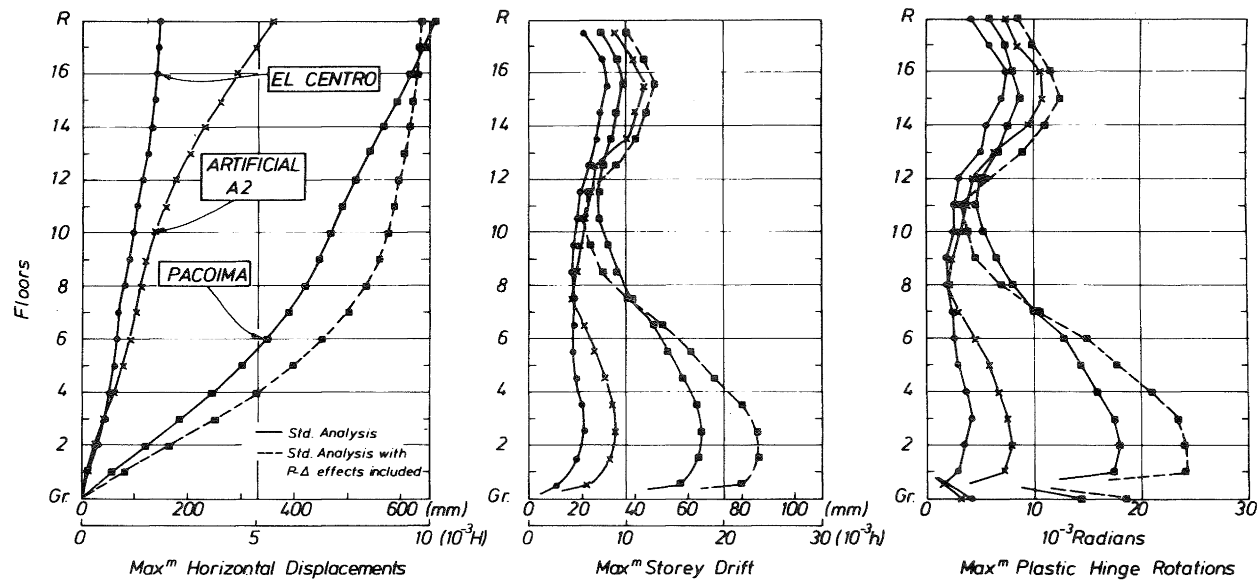
The horizontal displacement envelopes are illustrated in Figure 22 while the interstorey drifts are shown in Figure 23. The maximum plastic hinge rotations in the beams and the ground floor columns are shown in Figure 24.

From Figure 22, it can be seen that while the Pacoima earthquake would cause a different displacement profile to the El Centro and Artificial A2 earthquakes, the top floor deflection under the Pacoima earthquake is only marginally above the code allowable maximum. Surprisingly, the inclusion of P-Delta effects actually causes a slight reduction in the displacement at the top floor. The drift envelopes shown in Figure 23 show that while the Pacoima earthquake causes the interstorey drifts to exceed the code values over the bottom half of the building, the A2 earthquake causes the code drift limit to be exceeded over the top third of the building. The inclusion of the P-Delta effects into the analysis using the Pacoima record causes a significant increase in the drifts over the lower half of the building and also over the top third of the building. While P-Delta effects increase the plastic hinge rotations in the beams in the bottom storeys, the rotations are still within achievable limits.

Figures 25 and 26 show the time histories of the deflections with and without the P-Delta effect under the Pacoima Dam excitation. It will be observed that although the top storey deflection is not significantly altered. the timewise distribution of deflection over the height of the building has changed.



Figs 19, 20 and 21 — 12 storey frame (Zone C): Deflections, drifts and plastic rotations.



Figs 22, 23 and 24 — 18 storey frame: Deflections, drifts and plastic rotations.

The small significance of the P-Delta effect on this frame is most probably accounted for by the conservatism of the New Zealand Loadings Code(6) for long period structures together with the observation from the 12 storey frame that increasing the strength is the most effective way of controlling interstorey drifts. This conservatism of the Loadings Code is increased by the lengthening of the fundamental natural period of the building due to inclusion of the P-Delta effect.

MEASURE OF P-DELTA EFFECT

In order to provide a measure of the magnitude of the P-Delta effect, the increase in maximum interstorey drift due to the P-Delta effect was plotted against the maximum interstorey drift for the analyses ignoring the P-Delta effect. The results for the 12 and 18 storey frames are shown in Figure 27. The 6 storey frame has not been included because of the insignificance of the P-Delta effect for this frame. It can be seen that if the maximum inter-storey drift without the P-Delta effect is less than about 0.015 of the storey height, then - for the 12 storey building - the inclusion of the P-Delta effect reduces the maximum response. This is most likely due to the fact that the P-Delta effect increases the natural period of the 12 storey building from 1.880 seconds to 1.905 seconds. If the non P-Delta drift was greater than about 0.02 of the storey height, then the drift appears to increase to greater than 150% of the drift without P-Delta effects - and also tends to exceed the limit of practical detailing for the plastic hinge rotation.

EFFECTS OF VERTICAL ACCELERATIONS OF THE P-DELTA RESPONSE

For low to moderate high-rise frames subjected to horizontal excitation, the effects of the vertical accelerations of the floor masses should not be of great consequence as the vertical floor displacements, and hence accelerations, are likely to be small. For these structures the overall deformations of the frames is largely a shearing deformation. As the slenderness of the structures increases, then there will be a more significant bending deformation of the frames leading to greater vertical displacements of the upper floors. This will possibly lead to significant changes in the column axial loads and hence to the lateral stiffness of the floors. At this stage the analyses would have to revert to the considerably more expensive non-linear geometric type analyses in that the stiffness would be required to be updated at virtually every time step.

Similar problems would be encountered if vertical components were included in the earthquake excitation. Consequences of these components are at present under investigation.

CONCLUSIONS

It was found that provided the maximum interstorey drifts due to the design earthquakes are not significantly greater than 0.01 of the storey height of the structure, then the P-Delta effects may be justifiably ignored. For greater interstorey drifts, the effect of the gravitational load leads to a rapidly increasing augmentation of the interstorey drifts and this quickly exceeds the ability of the structure to provide the necessary ductility.

It also appears, that for inelastic frames, increasing the strength rather than the stiffness offers the most effective means of controlling these increases in displacement.

From the structures studied in this report it is evident that for frames designed to the present level of seismic loading and with the present limit on interstorey drift, the P-Delta effect is of no consequence as far as the present design procedures are concerned.

It is the opinion of the authors that future inelastic time-history analyses of multi-storey frames should include the P-Delta effect. This effect is always present in the real structures and consequently any analyses not including P-Delta are really artificial and incomplete. Further, for most structures there will be a negligible increase in the computational effort if the simplification used in this report is used, i.e. the initial gravitational loads in the columns are used together with the instantaneous drifts.

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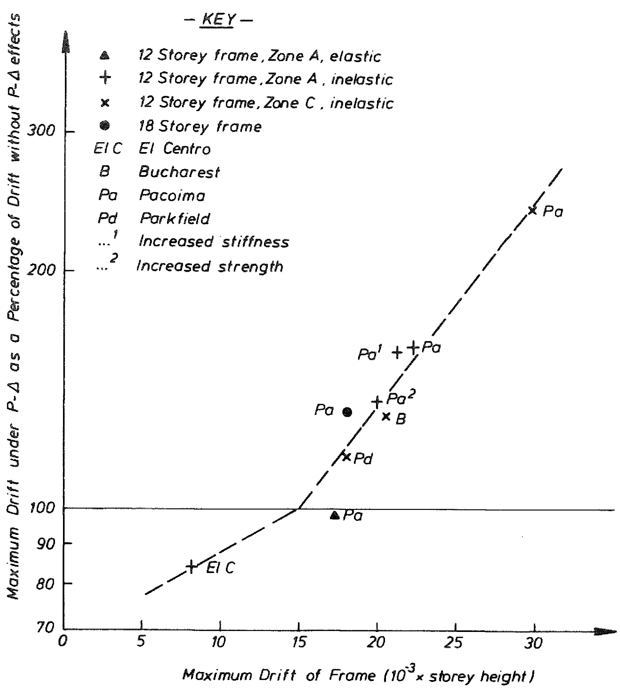

Fig. 27 – Increase in interstorey drift caused by the P–Delta effect.

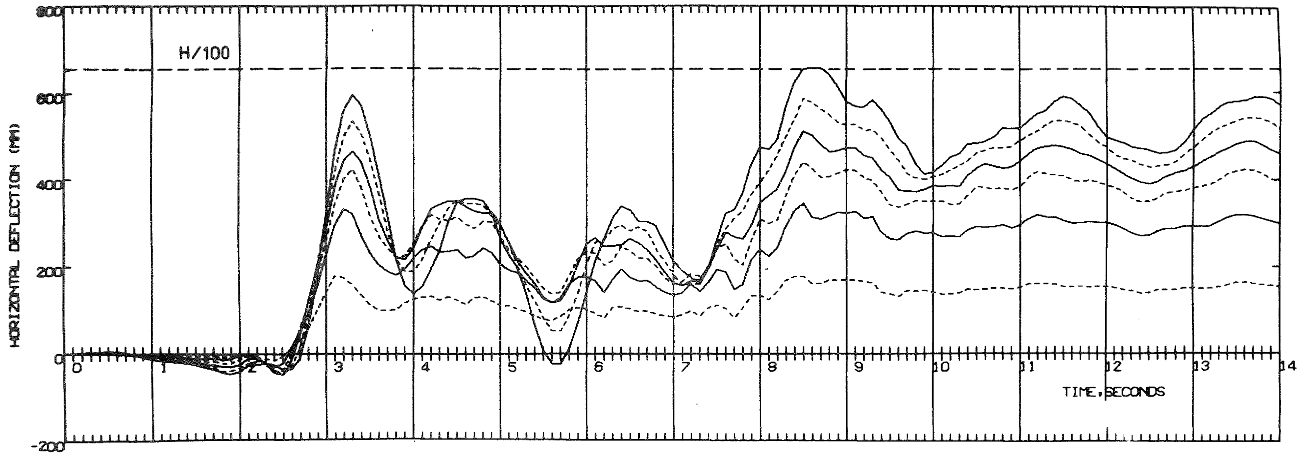

Fig. 25 – 18 storey frame: displacement history for every third floor, Pacoima earthquake.

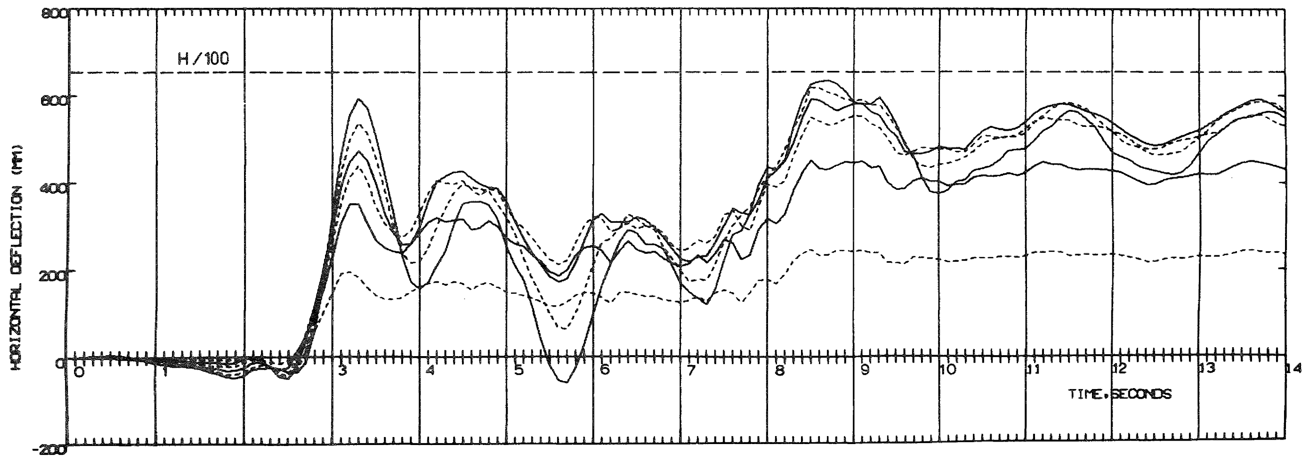

Fig. 26 – 18 storey frame with P–Delta: displacement history for every third floor, Pacoima earthquake.