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THE USE OF CRITICAL STATE SOIL MECHANICS TO CHARACTERISE CHRISTCHURCH SOIL IN RELATION TO LIQUEFACTION SUSCEPTIBILITY

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ABSTRACT

The majority of current procedures used to deduce liquefaction potential of soils rely on empirical methods. These methods have been proven to work in the past, but these methods are known to overestimate the liquefaction potential in certain regions of Christchurch due to a whole range of factors, and the theoretical basis behind these methods cannot be explained scientifically. Critical state soil mechanics theory was chosen to provide an explanation for the soil's behaviour during the undrained shearing. Soils from two sites in Christchurch were characterised at regular intervals for the critical layers and tested for the critical state lines (CSL). Various models and relationships were then used to predict the CSL and compared with the actual CSL. However none of the methods used managed to predict the CSL accurately, and a separate Christchurch exclusive relationship was proposed. The resultant state parameter values could be obtained from shear-wave velocity plots and were then developed into cyclic resistance ratios (CRR). These were subsequently compared with cyclic stress ratios (CSR) from recent Christchurch earthquakes to obtain the factor of safety. This CSL-based approach was compared with other empirical methods and was shown to yield a favourable relationship with visual observations at the sites' locations following the earthquake.

INTRODUCTION

The recent 2010 Darfield and 2011 Christchurch earthquakes caused widespread destruction in the Christchurch region. These events have raised interest in the region's geological setting, which has been found to be unique due to its high fines content and loose soil fabric. This has prompted a rise in research in the area linked with the geological setting, liquefaction features and future liquefaction prediction [1, 2]. However, there is still a lack of information of Christchurch's soil characteristics and behaviour with regards to soils often referred to in other empirical databases.

The majority of current procedures used to deduce the liquefaction potential of soils rely on empirical methods, such as those prescribed by Robertson and Wride [3] and Kayen et al. [4]. These methods have been proven to work in the past, but these are solely reliant on historical case data. While Boulanger and Idriss [5] has begun to incorporate some Christchurch case historical data into their empirical relationships, the majority of methods still tend to overestimate the liquefaction potential in regions to the north, west and south of Christchurch. This is partly due to, among many factors, the partial saturation of up to 6m below the water table in historical swamp areas, limitations of the cone penetration tests in characterising subsoil conditions accurately and higher liquefaction resistances due to the unique nature of Christchurch soils with its high fines content [6]. A scientific explanation of the soil's behaviour is thus required to illustrate the mechanics of the soil behaviour in relation to the unique properties of the Christchurch soil for the purpose of obtaining a method specific to the Christchurch region.

Current literature lacks the use of a scientific model to fully explain the onset of liquefaction. The critical state soil mechanics model proposed by Schofield and Wroth [7] provides a justifiable explanation for the soil's behaviour during the undrained shearing process caused by an earthquake. Thus, the use of critical state soil mechanics model to present a different approach to liquefaction other than empirical models, specifically for Christchurch soil, is attempted. The approach included the incorporation of the state parameter as a means of deducing how liquefiable the soil was, together with other relationships, such as that by Hardin and Richart [8], to deduce a relationship between shear-wave velocity, void ratio and confining pressure.

Various relationships were sought, utilising the soil's intrinsic properties and its link with critical void ratio and confining pressure. The hypoplastic model proposed by Herle and Gudehus [9] and void ratio range relationships proposed by Cubrinovski and Ishihara [10] were examined for this purpose. Penetrometer readings were also investigated to obtain the in-situ state at various depths. Basic characterisation, oedometer and triaxial tests were carried out in order to gain a greater understanding of Christchurch soil.

OBJECTIVE AND SCOPE

The aim of this paper is to present a better understanding of soils of the Christchurch region, including a comparison between Christchurch soils and soils belonging to existing empirical databases. The behaviour of Christchurch soils when loaded in an undrained scenario, which is similar to that of liquefaction, is studied and compared to that predicted by empirical models. It is hoped that the liquefaction characterisation of the soil layers from two sites in Christchurch could be obtained via the use of critical state soil mechanics for the purpose of evaluating their susceptibility to liquefaction. The viability of other methods with regards to liquefaction evaluation is also sought.

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Due to time and resource limitations, only two sites are assessed. However, these sites belong to two different geological formations, the Springston Formation and the Christchurch Formation, and are located in two completely different districts in Christchurch. The soils are also examined from various viewpoints, including critical state soil mechanics and empirical methods and research carried out in previous studies in the region (i.e. until mid-2015 when this research was conducted) are also accounted for.

CHRISTCHURCH GEOLOGY AND SEISMICITY

The city of Christchurch is built upon predominantly alluvial soils, and is interwoven with rivers flowing from the Southern Alps to the sea. The western and greater Christchurch regions are known to have a surface geology of the Springston Formation. These soils, the majority of which are overbank silt deposits, are known to be loosely placed due to the low energy deposition nature from the glacier-fed rivers and are fluvial deposits [11]. The eastern margin of the city is known to consist of Christchurch Formation sediments and is known to be denser and less prone to liquefaction compared to the Springston Formation [1, 11]. The Riccarton Gravel Formation underlies the Springston Formation and the Christchurch Formation, and is 300-400m deep. The groundwater table is known to be very high in the Canterbury region whereby artesian aquifers underlay Christchurch, and the groundwater surface is approximately 2 to 3m below the ground surface in the west and 0 to 2m in the eastern and central areas of the city.

The first earthquake, which brought to attention Christchurch's susceptibility to liquefaction, was the 2010 Darfield earthquake which had a magnitude of 7.1 Due to the distance to the epicentre, recorded peak ground accelerations (PGA) of 0.15-0.3g were recorded in the city, and the resultant of this earthquake did not cause too much damage to Christchurch [2]. The ensuing 2011 Christchurch earthquake, possessed a magnitude of 6.3 and caused more damage to the city, even resulting in 185 casualties. This was due to the hypocentre being shallower to the surface and less than 10 km away from the city centre. The resultant PGAs recorded from the M6.3 earthquake were reportedly three to four times greater within the city centre compared to the M7.1 earthquake, with an exceedingly strong vertical component [12].

CRITICAL STATE SOIL MECHANICS

The critical state of soil can be defined as the ultimate condition when plastic shearing of the soil continues to occur indefinitely without further changes in specific volume, mean effective stress or deviatoric stress. The soil can be said to behave as a "frictional fluid" and it is as if the material was a liquid under pressure [7]. This relationship can be exhibited with the following equation:

$$\frac{\partial p'}{\partial \varepsilon_q} = \frac{\partial q}{\partial \varepsilon_q} = \frac{\partial v}{\partial \varepsilon_q} = 0 \tag{1}$$

where p' is the effective stress, q is the deviatoric stress, v is the specific volume, and ε_q is the shear strain of the soil. The critical state of the soil is known to be the ultimate end point when a soil is sheared.

A series of critical state points will form the critical state line (CSL) for the specific soil, and can also be represented as a linear relationship on a void ratio vs log effective stress (e-log p') plot as shown in Figure 1. The linear relationship can be expressed as:

$$e_c = \Gamma - \lambda \log p'_c \tag{2}$$

where e_c is the void ratio at the critical state, Γ is the intercept of the CSL with the vertical axis, λ is the gradient of the CSL, and p'_c is the mean effective stress at the critical state. The equation can be visualised from the figure, where e_c represents any of the void ratios along the CSL with the corresponding p'_c. The CSL parameters, Γ and λ, can also be visualised in the figure. The significance of the CSL on the e-log p' plot is that it distinguishes what would be described as "loose" soils from "dense" soils. The term "loose" is used because when the soil is sheared, it will rearrange into a denser arrangement, and vice versa with the "dense" soil. The soil can also be described as contractive when it is "loose" and lies to the right of the CSL and dilative when it is "dense" and lies to the left of the CSL due to the nature of the soil when it is sheared.

Contractive and dilative behaviour of the soil can be determined by the state parameter (ψ) of the soil, which is best exhibited with the following equation:

$$\psi = e - e_c \tag{3}$$

where e is the current void ratio of the soil. A positive state parameter indicates that the soil is located to the right of the CSL and can be associated with contractive behaviour and vice versa, i.e. negative state parameter for dilative behaviour.

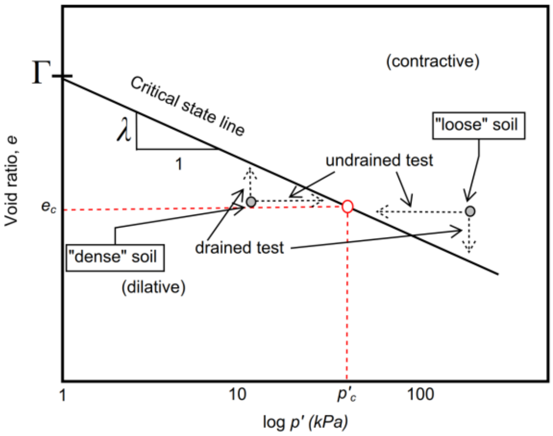

Figure 1: Representation of Critical State Line on the e – log p' plot [13].

LIQUEFACTION

The term liquefaction is used to describe the behaviour at which soil begins to act as a liquid or "frictional fluid" as described by Schofield and Wroth [7]. Seed et al. [14] explained it with a simple statement: liquefaction occurs when the excess pore water pressure generated equals total stress, i.e. when the effective stress equals zero. This generation of excess pore water pressure becomes significant if the soil layer is subjected to an undrained boundary condition and can be brought upon by two methods: through statically-induced stresses or through cyclic-induced stresses. The difference between these two methods is the way in which plastic strains are generated. In static liquefaction, the soil is needed to be in a "loose" state, i.e. it requires a state parameter greater than zero, and thus exhibit contractive behaviour. In exhibiting contractive behaviour, the soil will generate excess pore water pressure when shear stress is applied to it, which will then exert a stress against the soil skeleton, reducing the contact pressure between soil particles and thus liquefying. In the case of cyclic liquefaction, the plastic strain on the soil is generated through the densification process brought on by cyclic stress changes, which tend to consolidate the soil particles further.

Unlike static liquefaction, cyclic liquefaction affects all types of soil, regardless of denseness and cohesiveness.

EXISTING EMPIRICAL METHODS

Until now, the characterisation and profiling of in-situ soils for liquefaction evaluation purposes have been mainly carried out using two methods: the expensive and time-consuming method of taking high-quality undisturbed samples for laboratory tests, or the relatively inexpensive and time-efficient method of penetration tests. The methods used to obtain undisturbed samples of soil have been proven to be satisfactory, but the process takes a lot of time and is prone to many factors which may affect the quality of the samples. Also, determining the strata and liquefaction potential of a soil layer many meters thick would be an almost impossible task. This leaves the penetration tests as a more viable method, at least for many conventional projects.

In-Situ Tests

The simple and inexpensive nature of in-situ penetrometer tests, such as standard penetration tests (SPT) and cone penetration tests (CPT), allow a large number of these tests to be carried out rapidly, thus being able to map out the overall soil strata and variability of ground properties for a given site. The rapid and inexpensive nature of penetrometer tests has outweighed their disadvantages and penetrometer tests are used widely in low-risk projects and in the initial estimation stages of high-risk projects [3].

Another alternative to these penetrometer tests which measure the soil resistance instead of the property of the soil is to obtain the shear wave velocity, V_s, of the soil using various methods, including a seismic dilatometer test (sDMT). These shear wave velocity tests measure a fundamental property of the soil, which is preferable to the SPT and CPT tests [4]. The shear wave velocity is directly related to the small strain shear modulus of the soil (G_{max}):

$$G_{max} = \rho V_s^2 \tag{4}$$

where ρ is the soil's bulk density. These penetrometer readings and shear wave velocity readings are then correlated to liquefaction potential using empirical methods. These empirical methods, originating from the work by Seed and Idriss [15], evaluate the likelihood of liquefaction occurring in a deterministic manner. They use the "factor of safety" concept which compared the cyclic shear stress ratio (CSR), which is the induced external shear force on a body of soil, to the cyclic resistance ratio (CRR) which is the maximum cyclic shear ratio the body of soil can withstand without undergoing liquefaction. CSR is typically evaluated empirically while the CRR is estimated using historical field data on liquefiable and non-liquefiable soils. The simplified procedure by Seed and Idriss [15] provided the basis for many existing liquefaction evaluation procedures [3, 4, 16].

The use of shear wave velocity for this study has advantages over penetrometer readings. One of the important advantages is that shear wave velocity-based methods are less affected by problems associated with high fines contents, which is present in Christchurch soils. This is due to the shear wave velocity of soil being less sensitive to fines and only requiring a minor correction [4]. Also, both shear wave velocity and liquefaction resistance are influenced by the same factors, including void ratio, confining stress and stress history [16].

In this paper, the empirical methods used are the CPT-based methods by Robertson and Wride [3], Boulanger and Idriss [5] and Moss et al. [17], and the shear wave velocity-based method by Kayen et al. [4].

Liquefaction Potential Index and Liquefaction Severity Number

The Liquefaction Potential Index (LPI) is a parameter developed by Iwasaki et al. [18] to express the severity of liquefaction-related damage at a site in relation to a specific earthquake. The LPI takes into account the depth of the liquefiable layers, the proximity of the liquefiable layers to the surface and the factor of safety (FoS) of the liquefiable layers.

The LPI typically ranges from a value of 0, for sites with no liquefaction potential, to a value of 100, for sites with FoS equal to zero over the entire 20m depth range [18]. An LPI value of approximately 5 would indicate the event of sand boils to occur, while an LPI greater than 12 would indicate the potential of lateral spreading to occur [19]. The LPI acts as an extra step when performing empirical methods as it adds an additional dimension to the liquefaction potential analysis. However it has to be noted that even though a soil column may indicate a low LPI reading, the corresponding area may still show liquefaction manifestations on the surface if the critical layer covers a very large surface area [20]. The opposite also applies in that a soil column having a high LPI reading may correspond to the site actually not showing liquefaction manifestation at all if the surface area of the critical layer is limited.

An alternative to the LPI would be the Liquefaction Severity Number (LSN) proposed by Tonkin and Taylor [21]. The LSN places a large emphasis on the damaging effects of shallow liquefaction, as opposed to deeper liquefaction. The LSN utilises the potential volumetric strain and settlement of individual soil layers, and places a weighting on it depending on the proximity to the surface. An LSN value less than 20 would indicate minor liquefaction surface manifestation, while a value from 20 to 50 would indicate moderate liquefaction surface manifestation and LSN values larger than 50 would indicate major liquefaction surface manifestation.

SHEAR WAVE VELOCITY, VOID RATIO AND CONFINING PRESSURE

The shear-wave velocity in a soil medium has been found to be dependent solely on in-situ void ratio, the effective confining pressure of the soil and the intrinsic properties of the soil, such as angularity, grading curve and fines content [8]. Unless the confining pressure is at a very high level such that grain crushing begins to occur and the intrinsic properties of the soil begin to be modified, the shear-wave velocity can be related to just the void ratio and effective confining pressure of the soil. It is possible to then predict the state parameter of the soil using instruments which measure shear-wave velocity in laboratories or on site. The relationships which Hardin and Richart [8] produced were based on the rounded Ottawa sand and the more angular crushed Quartz sand. They obtained different relationships for each of these sands and attributed the difference to the angularity and composition of the soil. However, when the shear wave velocities were normalised it is shown that the multiple relationships converge into a single relationship, as shown in Figure 2. The normalisation equation is given in Equation (5):

$$V_{s(n)} = V_s\left(\frac{P_a}{\sigma_v'}\right)^{0.25} \tag{5}$$

where P_a is the reference atmospheric pressure (98 kPa) and $\sigma_v{'}$ is the vertical effective stress. Robertson et al. [22] and Robertson and Fear [23] also carried out shear wave velocity tests on other soils such as Alaska sand and Syncrude sand, and their results were found to align with Hardin and Richart's [8] relationships, shown in Figure 3. The data plots are from Robertson and Fear's [23] study, and the relationships from

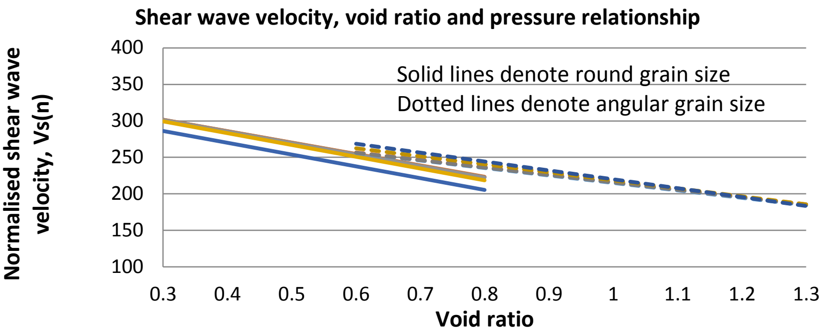

Figure 2: Normalised shear wave velocity relationship of Hardin and Richart [8].

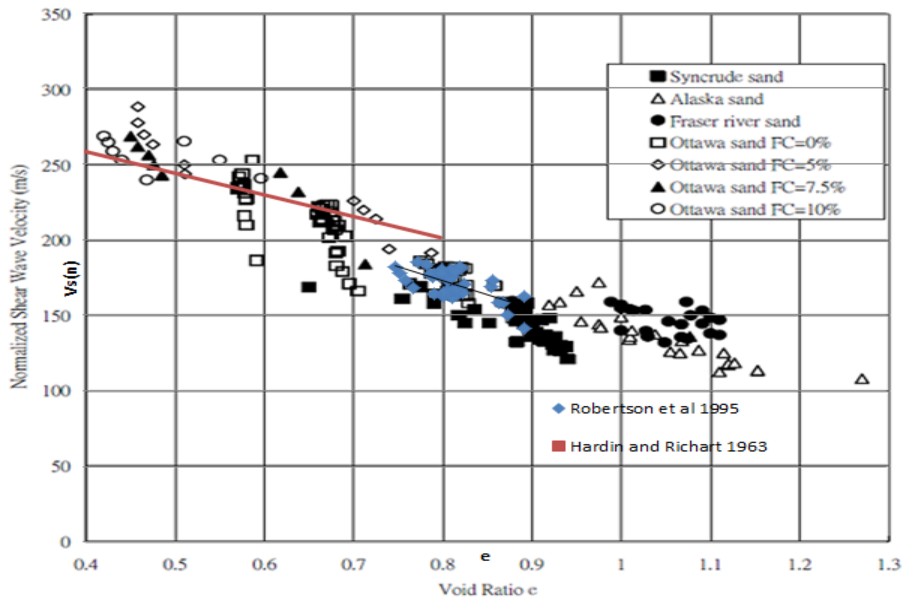

Figure 3: Multiple relationships between void ratio and normalised shear wave velocity [24].

the other two studies are shown to superimpose well with these data plots.

The relationship between void ratio and pressure is more complex than that of void ratio and shear-wave velocity and is often represented with a model. Models, such as the Cam-Clay model, rely on elastoplasticity, a concept which is a conjunction of elastic behaviour within a certain yield boundary and plastic behaviour outside the boundary [25]. Niemunis [26] stated that elastoplastic models lack the ability to facilitate the spontaneous localisation of deformation, which is known to be a common occurrence in soil. A different model is the hypoplastic model, which is a nonlinear constitutive theory of granular materials which states that neither stress nor energy is recovered after any strain cycle.

When compared with elastoplasticity, hypoplasticity results in permanent deformation of the soil when subjected to any magnitude of stress. This falls in line with the fact that soil behaviour is nonlinear and mostly irrecoverable during compression and shearing. Hypoplasticity assumes that the grains of a soil are combined to form a "simple granular skeleton" of which the grains are permanent and any deformation in this granular skeleton is due to rearrangement of the individual grains. Certain factors are considered to be negligible such as any compression, abrasion or crushing of the grain, or surficial effects between grains, such as capillarity, cementation or osmotic pressures [27]. Further information and details about the hypoplastic mechanism can be obtained from the work by Herle and Gudehus [9].

CHRISTCHURCH SITES AND SOIL PROPERTIES

The soils used in testing were sampled from St George's Hospital (SGH) and Pumpstation 15 (PS15). Based on visual observations, the SGH site underwent only minor liquefaction while severe liquefaction occurred in PS15 site. The SGH site is located within the Springston formation in the suburb of Merivale, to the northwest of Christchurch CBD. St George's Hospital sustained damage during the February 2011 earthquake resulting in the closure and demolition of one of the main hospital wings. Sand boils and subterranean sand ejecta were observed at this site.

Pumpstation 15, located in Woolston, collects wastewater from that region and pumps it to the wastewater treatment plant in Bromley. Pumpstation 15 sustained significant damage in February 2011, and the sand ejecta observed was more severe compared to those at St George's Hospital. Lateral spreading was absent at both sites, as the ground was fairly level and far from the river.

Both these sites have important structures in Christchurch's infrastructure, and are thus a necessity in the daily running of the city. Both these sites were specifically chosen due to their different locations on two of Christchurch's geological formations, which would thus give a better representation of Christchurch soils. The level of liquefaction observed at both sites were different as well, with PS15 experiencing more severe manifestation, and the difference between liquefaction severities could thus be examined. Borelogs extending through

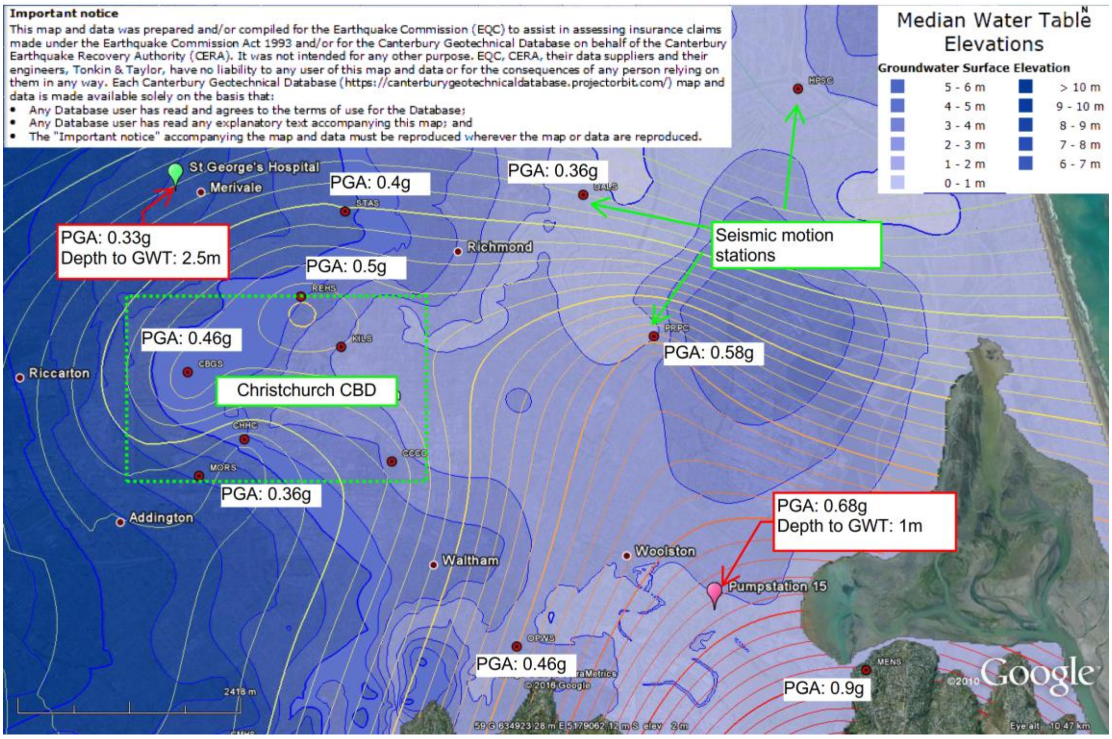

Figure 4: PGA contours and depth to the water table for Christchurch for the February 2011 earthquake [28].

the upper 20m layer were available from both sites, allowing the critical layer to be ascertained with greater confidence.

The water table and PGA contours for the February 2011 Christchurch earthquake, as well as the locations of the seismic motion stations are shown in Figure 4. The St George's Hospital site had a deeper water table at 2.5m compared to the Pumpstation 15 site, as the overall trend in Christchurch City was a shallower water table to the east; it was noted that Pumpstation 15 was closer to various streams as well. This water table recording was taken the day before the earthquake and obtained from the Canterbury Geotechnical Database [28]. The PGA obtained at SGH site was considerably lower than that at PS15 site due to the distance from the epicentre of the earthquake which was located to the southeast of Christchurch. The locations of other seismic motion stations from where the PGA contours are based are also shown in the figure.

TESTING METHODOLOGY

The testing strategy used during the study was to first ascertain the basic characteristics of the soil with preliminary tests. Once the characteristics of the soils were known, appropriate adjustments could be made for the oedometer and triaxial tests, i.e. formation of samples, and the feasibility of certain tests could be ascertained based on the equipment available. The results from the preliminary tests were also compared with the readings of empirical methods, as well as the CPT and V_s readings at various depths.

The critical state lines (CSL) of the various soils at different depths were then found, and a means of correlating the position (Γ) and slope (λ) of the CSL with the intrinsic properties of the soil was sought. Using the theory of critical state soil mechanics, the in-situ void ratio and corresponding critical void ratio was obtained from the shear wave velocity readings performed at the site. The state parameter and CRR of the soil could then be obtained using existing soil relationships [29].

Preliminary Tests

The soil samples were obtained via sonic core drilling carried out on BH10b at SGH and BH1 at PS15 by McMillan Drilling Services. Details of the logs are obtainable from the Canterbury Geotechnical Database [28] under Location ID 76071 to 76075. Preliminary tests were carried out only on the critical layer of the soil strata due to time and equipment restriction. Soil was sampled at every 1m for the SGH site and at every 2m for the PS15. Index property tests were conducted to determine the grain size distribution, fines content (F_c), solid density (G_s), maximum and minimum void ratios (e_{max} and e_{min}, respectively) and constant volume friction angle (ϕ_{cv}). The tests were carried out according to NZS4402 [30] where possible, but variations had to be carried out due to equipment and sample restrictions.

Oedometer Tests

The oedometer tests were required in order to ascertain the parameters h_s and n in the hypoplastic model proposed by Herle and Gudehus [9]. The parameter h_s, which was used to set a reference pressure, and n, the pressure sensitivity of the grain skeleton at the reference pressure, were obtained through the one-dimensional compression curves. The conditions for this compression tests were for the soil to be as wet and loose as possible [9].

Consolidated Undrained Compression Triaxial Tests

The consolidated undrained (CU) compression triaxial test was chosen to obtain the soil's critical state. The test procedure adopted was based on ASTM D4767 [31], and the samples were prepared using moist tamping method. Moist tamping was used even though fluvial deposition would likely best re-enact the soil fabric observed in natural soils as moist tamping would allow the sample to be formed with very high void ratios. This in turn allows the CSL to be obtained at very low confining pressures [32]. Moist tamping also avoids segregation between fines and large particles and allows global sample density to be controlled. The tests conducted had varying void ratios and consolidation pressures to obtain the CSL, using soils sampled from different depths at the two sites.

TEST RESULTS

The results of the preliminary tests on samples from SGH site have been summarised in Table 1, while the grain size distribution curves are shown in Figure 5. The soils tested were from 3-9m depth as this layer was identified to be critical. The majority of the soil was observed to have extremely high fines content F_c (grain size diameter < 75μm), with the lowest F_c observed being around 37% at the 5m depth. The majority of the soil would thus almost certainly have a higher F_c value than the transition fines content (TFC) which Thevanayagam and Martin [33] have prescribed. This indicates that the soil is more likely to behave as a fines material rather than as sand. The predominant composition of fines in the soil may also cause some inaccuracies in other index properties of the soil.

The layers at 7-9m in SGH site were not considered any further in this study due to the extremely high F_c of those layers (> 95%). The plasticity index (PI) of these soils was found to be 7.7, which was deduced to be sufficiently high enough such that the soil would behave in a clay-like manner [34] and unlikely to liquefy.

The soils at PS15 site were observed to be larger in grain size compared to those at SGH site, and more cohesionless in nature. The index properties are summarised in Table 2 while the grain size distribution curves are shown in Figure 6. The soil tested originated from depths of 4-16m. The PI was not determined for the PS15 site as the grains of the soil was found to be visibly and assuredly non-plastic.

The majority of the soil was seen to have F_c of approximately 10-14% with the exception of the soil at the 8m layer which had F_c = 35.4%. This is apparent in Figure 6 whereby the shape and position of the curves are all very similar. This soil can be characterised as silty sand, and showed relatively uniform properties across all layers when compared with the soil at SGH site.

Comparison with Existing Soil Database

The soil properties obtained from the index property tests on soils from the two sites were compared with Cubrinovski and Ishihara's [35] soil database. The test procedures used for soils in the said database range from the methods described in ASTM and JGS procedures, with a large proportion based on non-standard procedures. This may influence the comparison between the Christchurch soil and the existing soil database, but as observed in Figure 7, the ratio of the maximum and minimum void ratios of the soils from the two sites fitted very well. The data from Rees [32] was also included for comparison purposes.

Cubrinovski and Ishihara [35] placed a lot of emphasis on a soil parameter they have conceived, the void ratio range (e_{max} – e_{min}), as they claimed that the void ratio range is affected by a lot of the other soil properties, including void ratios, fines content, angularity and mean grain size. Figure 8 shows that the values of void ratio range and maximum void ratio values obtained for the two soils fit the general trend, and together with the Rees [32] data, validates the void ratio range values of the Christchurch soil.

Table 1: Index properties of the soils at St George's Hospital site.

Depth (m)	e_{max}	e_{min}	*Void ratio range*	*Fines Content (%)*	ϕ_{cv} (°)	*Specific Gravity*
3	1.35	0.84	0.51	77.3	38.4	2.67
4	1.21	0.60	0.61	50.0	36.3	2.66
5	1.17	0.68	0.49	36.6	32.3	2.65
6	1.50	0.81	0.69	86.8	35.5	2.68
7	1.79	0.97	0.82	96.9	37.7	2.69
8	2.47	1.15	1.32	99.2	43.3	2.67
9	2.92	1.22	1.70	99.8	47.3	2.72

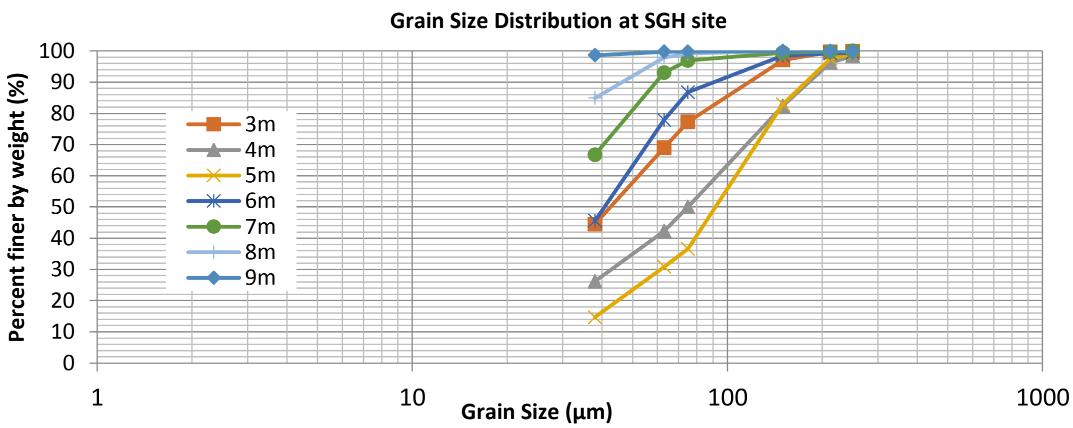

Figure 5: Grain size distribution curves of soils from St George's Hospital site.

Table 2: Index properties of soils at Pumpstation 15 site.

Depth (m)	e_{max}	e_{min}	*Void ratio range*	*Fines Content (%)*	ϕ_{cv} (°)	*Specific Gravity*
4	1.11	0.71	0.40	11.8	35.5	2.69
6	1.12	0.70	0.42	12.8	33.3	2.72
8	1.17	0.73	0.44	35.4	34.8	2.72
10	1.17	0.76	0.41	10.5	35.7	2.72
12	1.19	0.75	0.44	13.6	33.7	2.69
14	1.10	0.73	0.37	10.0	33.4	2.69
16	1.03	0.66	0.37	11.9	36.7	2.68

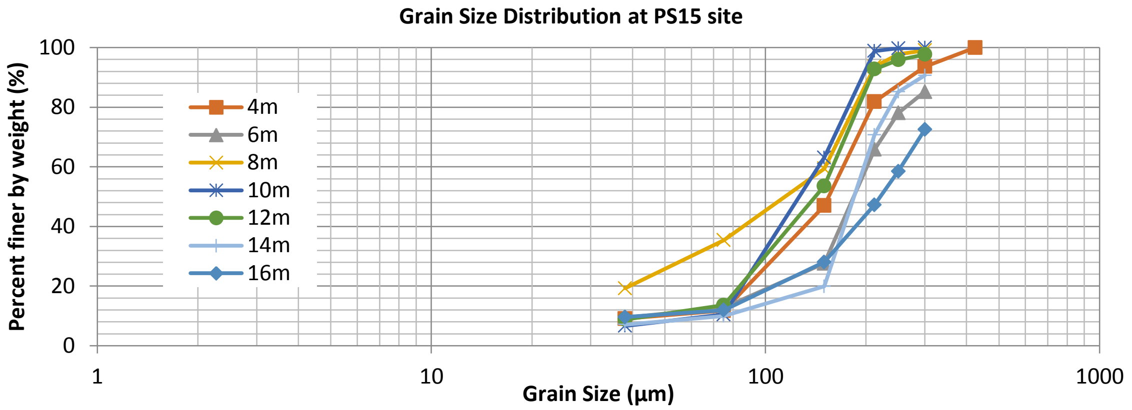

Figure 6: Grain size distribution curves of soils at Pumpstation 15 site.

However, the fines content to void ratio range relationships for the Christchurch soils, shown in Figure 9, do not match the soil database. For the specified F_c value, the soils from PS15 and SGH sites were both shown to have a consistently lower void ratio ranges than the other soils in the database and this could be an indicator that the grain characteristics of the Christchurch soil is unique and has a different relationship with the void ratio range. Figure 10 reiterated this observation as the mean grain size and void ratio range of the Christchurch soils again did not match the soil database, with the Christchurch soils showing consistently lower void ratio range outside the confidence interval. This evidence points towards Christchurch soils being unique with respect to the fines content and the mean grain size, and may not be compatible with certain liquefaction evaluation methods. This idea has also been emphasised by the plotting of Rees [32] data on Figures 9 and 10, which has been shown to also plot outside the confidence intervals.

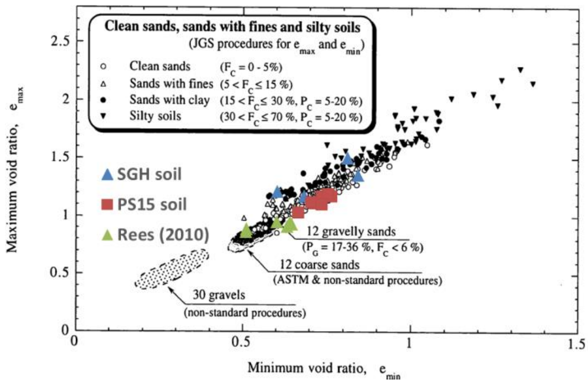

Figure 7: Comparison between maximum and minimum void ratio of Christchurch soils with Cubrinovski and Ishihara [35] database.

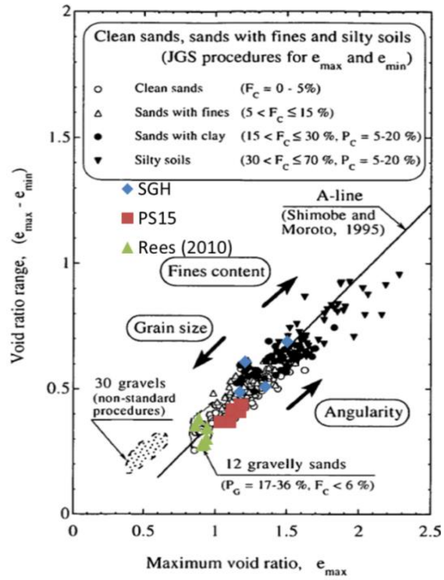

Figure 8: Comparison between void ratio range and maximum void ratio of Christchurch soils with Cubrinovski and Ishihara [35] database.

Critical State Line of Christchurch Soil

The next objective was to obtain a means of finding the Critical State Line (CSL) using a range of different methods. The first step was to obtain the actual CSL at various depths using the results from the CU triaxial tests. The soil properties at different depths were examined and used to explain the differences in the CSL obtained. Existing models and relationships used to predict the CSL were then compared to validate the applicability of these models to the soils tested.

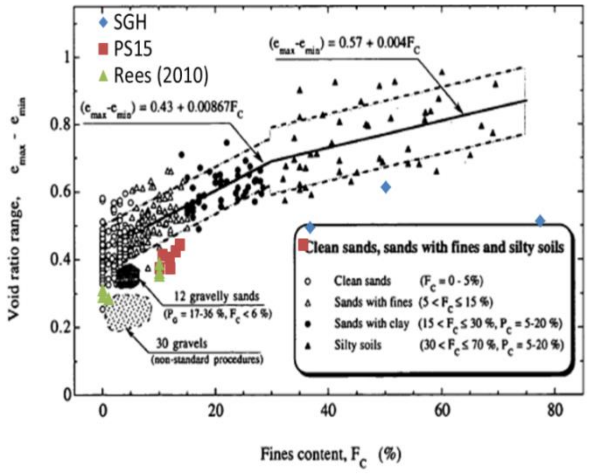

Figure 9: Comparison between void ratio range and fines content of Christchurch soils with Cubrinovski and Ishihara [35] database.

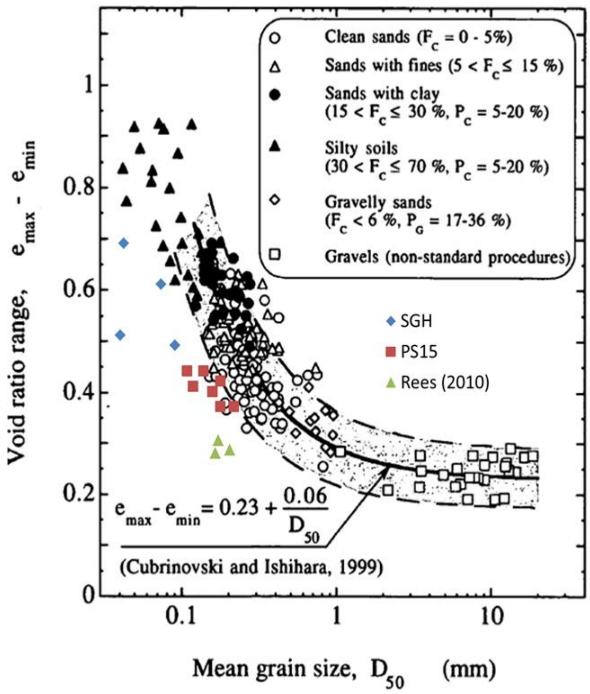

Figure 10: Comparison between void ratio range and mean grain size of Christchurch soils with Cubrinovski and Ishihara [35] database.

Laboratory Tested Critical State Line

The CSL of the soils obtained from various depths at both SGH and PS15 sites was obtained using the CU triaxial test results. The CSL for each soil layer was determined using five critical state points, i.e. at four different consolidation pressures (50 kPa, 100 kPa, 150 kPa and 200 kPa) and the e_{max} of the soil as the fifth point. It is noted that the use of e_{max} as the fifth point for the CSL was limited to soils with $F_c \leq 50\%$, as soils with $F_c > 50\%$ did not yield an e_{max} which matched the other four critical state points due to limitations of the void ratio determination procedures [30]. This could also be due to the behaviour of the soil being governed by the fines of the soil, as the transitional fines content (TFC) of the soil has been exceeded [33].

It was observed that the critical state points followed a linear trend with very little scatter around the CSL. The positions (Γ) and the slopes (λ) of the CSL were then compiled and are shown in Table 3. The Γ of the CSL has been set at a reference pressure of 1kPa and a 1:1 ratio between e_{max} and Γ at 1kPa has been validated [29].

Table 3: Summary of CSL positions and slopes.

Site Location	Depth (m)	λ	Γ
St George's Hospital	3	0.14	1.05
	4	0.23	1.21
	5	0.15	1.18
	6	0.19	1.18
Pumpstation 15	4	0.13	1.10
	6	0.13	1.12
	8	0.14	1.19
	10	0.12	1.17
	12	0.16	1.19
	14	0.10	1.10
	16	0.10	1.04

Critical State Line Prediction Using Void Ratio Range

Cubrinovski and Ishihara [10] produced a relationship between Γ and void ratio range, and also λ and void ratio range. Using these two relationships, a linear CSL can be predicted for the various depths and the predicted Γ and λ for the soils at various depths are summarised in Table 4. It can be seen that the Γ and the λ of the PS15 soils did not have a large spread throughout the depths, which is due to the void ratio range being relatively constant, unlike those of SGH soils which have a large variance of Γ and λ. These results thus exemplify this method's dependence on void ratios, and are important when considering which standards to use to determine the void ratios.

Table 4: Predicted critical state line parameters using void ratio range.

Site Location	Depth (m)	Cubrinovski & Ishihara [10]	
		Γ	λ
St George's Hospital	3	1.19	0.11
	4	0.93	0.13
	5	1.03	0.10
	6	1.11	0.15
Pumpstation 15	4	1.05	0.04
	6	1.04	0.04
	8	1.07	0.05
	10	1.10	0.04
	12	1.09	0.05
	14	1.06	0.04
	16	0.99	0.04

The use of Cubrinovski and Ishihara's [10] relationships to obtain the CSL proved to be extremely easy, as it only required the e_{max} and e_{min} of the soil. This allows the triaxial testing stage to be skipped, which takes a relatively long time to perform. However, care must be taken with this quick approach because of possible inadequacies, especially when used for large-scale or important projects.

CSL Interpretation of Hypoplastic Model

The parameters required in the hypoplastic model were obtained from oedometer test results, and these are presented in Table 5. It is noted that sample preparation and disturbances has a large influence on h_s, and it is evident that there is a large spread in values between the different soil layers. The values were significantly different to the values obtained from various sands compiled by Herle and Gudehus [9]. This may be due to the different intrinsic soil properties, as the sands which Herle and Gudehus [9] reported were more granular in nature and had a much larger mean grain size, d_{50}. Limited research has been carried out in the application of the hypoplastic model to fine-grained and cohesionless soils [36], thus preventing the comparison of Christchurch soil values with other soils of similar properties.

Table 5: Hypoplastic model parameters from oedometer tests.

Site	*Depth(m)*	*n*	*h_s (MPa)*
St George's Hospital	3	0.57	20.3
	4	0.51	21.5
	5	0.43	80.2
	6	0.45	61.2
Pumpstation 15	4	0.37	539.8
	6	0.34	357.5
	8	0.53	32.4
	10	0.36	1753.8
	12	0.37	515.7
	14	0.42	316.2
	16	0.33	343.5

The use of the hypoplastic model to predict the CSL requires an input specific to the compressibility of the soil, making the model unique for different types of soil. This is different from the Cubrinovski and Ishihara's [10] method whereby they attempted to correlate the CSL with the void ratio, a value which captures the intrinsic property of the soil rather than the behaviour of the soil. This indicates that the hypoplastic model will yield a unique CSL for different types of soil, and therefore has a sound theoretical basis. However, the model requires frequent oedometer or triaxial testing at regular depth intervals if the soil composition is variable, making it very time consuming. The hypoplastic model parameters, h_s and n, can be assigned on a site-to-site basis if the soil in that area is known to be uniform and continuous; however, this process will still be time consuming as it would have to rely on other tests to confirm this. Another factor is that the hypoplastic model yields a curved CSL for the p' range considered which was from 50kPa to 200kPa. This is incompatible with the CSL obtained from the laboratory tests, which predicted a linear trend between 50 kPa and 200 kPa, i.e. the pressure range for the target soils. Note, however, that it is possible for the CSL to curve at larger pressure range, but that is outside the scope of this study.

Comparison between the Predicted and Actual CSL

The previous two methods described to predict the CSL of the Christchurch soils, i.e., Cubrinovski and Ishihara's [10] relationship and the hypoplastic model, are compared with the CSL obtained from the laboratory tests. Representative examples are shown in Figures 11 and 12. The hypoplastic model is seen to significantly and consistently overestimate the lab-obtained CSL regardless of site and depth; the magnitude of overestimation is also different for all the soil layers. In general, there is no obvious trend between the hypoplastic model's predicted CSL and the actual CSL.

Cubrinovski and Ishihara's [10] predicted CSL is seen to be closer to the lab-obtained CSL values, but is still significantly different, as the λ predicted was observed to be lower than the actual value. The Γ of the CSL predicted by Cubrinovski and Ishihara [10] relationships are also observed to be very random and no obvious trends between these predicted values and the actual CSL Γ and λ values are evident. It can thus be concluded that, based at least on the limited information available, neither of these methods are suitable to predict the CSL of the Christchurch soils used.

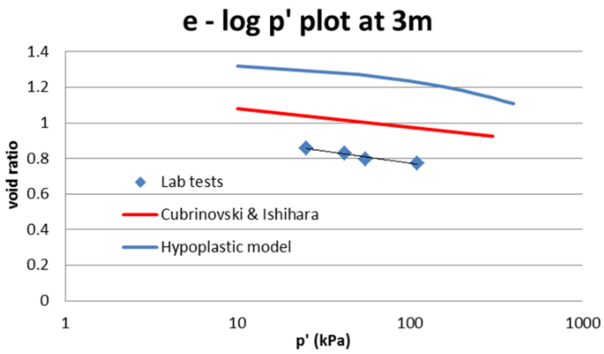

Figure 11: Comparison between actual and predicted CSL for St George's Hospital site.

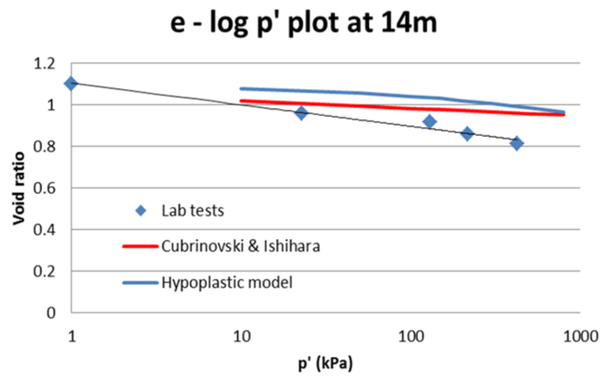

Figure 12: Comparison between actual and predicted CSL for Pumpstation 15 site.

Proposed Method to Predict CSL

It was desired that the parameters Γ and λ be obtained via a simple method without the need of going through complicated triaxial tests. The solution to this was to find a correlation between soil properties with the Γ and the λ. However, it was determined that only Christchurch soils would be used in the formulation of these relationships as these soils have been proven not to fit the current relationships based on the hypoplastic empirical formulations, and need to be approached separately.

Various relationships between soil properties and the CSL parameters of the soils at SGH and PS15 sites were examined to obtain a suitable correlation. A 1:1 ratio between e_{max} and Γ was validated, and this relationship is illustrated in Figure 13 obtained from Been and Jefferies [29]. The Γ obtained is solely based on the soil itself, and thus will not be complicated by soil data from other locations. The outliers in the figure originating from the SGH site can be attributed to the very high F_c of these soils, which were 77.3% and 86.8%. The F_c limit for the use of the 1:1 e_{max} to Γ assumption is 50%, as soils with a higher F_c will not fit the trend.

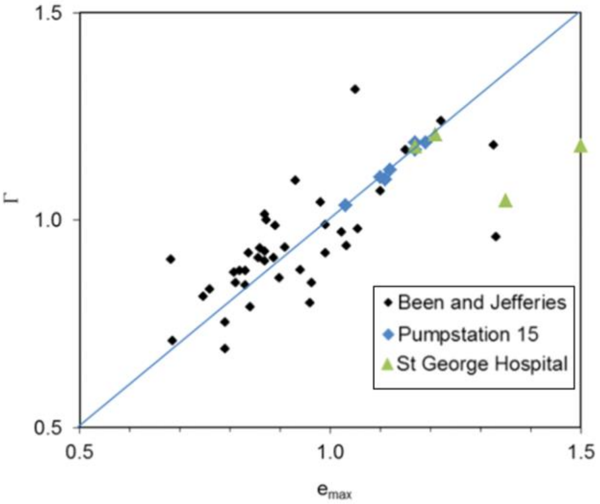

Figure 13: Relationship between Γ and $e_{max[}$ [29].

The value of λ was first determined based on correlation with F_c of the soil, as proposed by Bouckovalas et al. [37], and the results are shown in Figure 14a. The relationship between λ and F_c showed weak correlation, with an R^2 value of 0.35. The next relationship attempted was with log F_c, as shown in Figure 14b, which yielded a slightly higher R^2 value of 0.43; however, there is still a considerable amount of scatter in the relationship. The basis for expressing F_c in log scale is also unjustifiable as a small change in F_c at low F_c values does not necessarily result in a large change in λ. Been and Jefferies [29] indicated that the fines content is not enough to be a predictor for λ.

Another approach would be to use the void ratio range concept proposed by Cubrinovski and Ishihara [10], but with certain limitations to improve the accuracy. The soil database would be confined to the soil samples from the SGH and PS15 sites, and the λ – void ratio range relationship is shown in Figure 15. This is more favourable than using Cubrinovski and Ishihara's [10] original relationship as it is not limited to clean sands and sands with less than 36% fines. It is evident that there is far less scatter and a more reliable R^2 value of 0.75 was obtained. There is greater variability at a higher void ratio range in terms of obtaining the λ, which is similar to the trend observed by Cubrinovski and Ishihara [10].

LIQUEFACTION RESISTANCE OF CHRISTCHURCH SOILS

After determining the in-situ CSL of the soil at various depths for the two sites, the next stage would be the use of this critical state soil mechanics-based method to predict the liquefaction potential of the soil. This section underlines the route undertaken to convert the state parameter of the in-situ soil to a factor of safety value, which will then be compared with other empirical methods as well as with visual observations from the sites.

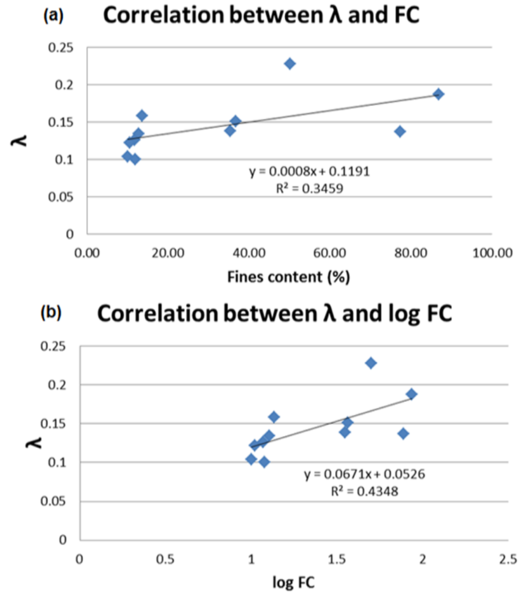

Figure 14: Correlation between (a) λ and F_C, (b) λ and log F_C for PS15 and SGH soils.

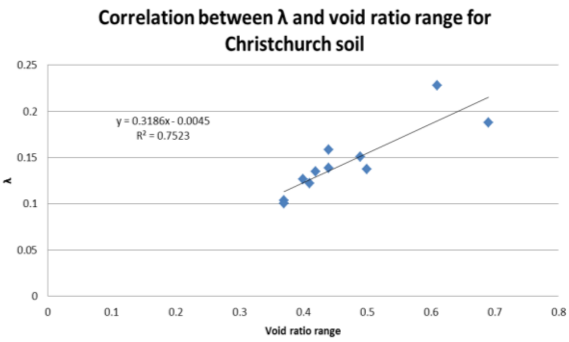

Figure 15: Correlation between λ and void ratio range for SGH and PS15 soils.

Factor of Safety via Critical State Soil Mechanics

To obtain Γ and λ of the CSL at various depths, the in-situ void ratio is needed, as indicated in the Hardin and Richart's [8] relationship. This allows the state parameter, ψ, to be ascertained. The next stage in determining the liquefaction potential of these soil layers would be to correlate this state parameter of the soil with the CRR. Been and Jefferies [29] performed extensive work in this field, and they obtained a relationship between the state parameter and the CRR. However the relationship they procured did not include any Christchurch-related sand, and when various Christchurch-related CRR – ψ data points were input into the graph they produced, a different relationship was obtained. The Christchurch data points were obtained from Fitzgerald Bridge and Lichfield Street in Christchurch [32], and Kaiapoi sand [38]. These two relationships can be seen in Figure 16. The coloured data plots represent the Christchurch soils while the black and white plots represent the data used in Been and Jefferies' study; it is shown in Figure 16 that a different CRR – ψ relationship is evident for the Christchurch soil. The dotted lines represent a one standard deviation confidence interval for both relationships, and it can be seen that the majority of data plots fall within the boundaries, indicating a satisfactory relationship.

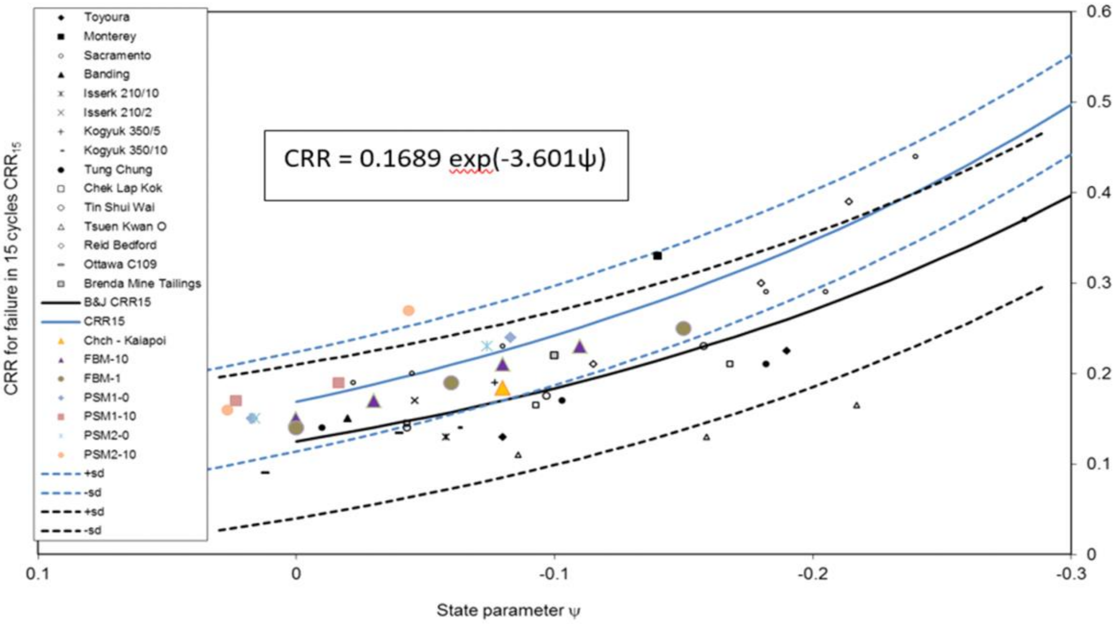

Figure 16: Comparison between CRR – ψ relationships for Christchurch soils and Been and Jefferies [29] data.

With the Christchurch-specific relationship shown in Figure 16, the $CRR_{7.5}$ of the PS15 and SGH soils can be obtained accurately. However, these $CRR_{7.5}$ values obtained from laboratory tests, assumed to represent a M7.5 earthquake, are different from the CRR in the field, and thus correction factors for earthquake magnitude, overburden stress and the initial static stress need to be applied to this $CRR_{7.5}$ value, represented by K_M, K_σ and K_α, respectively. The relationship between $CRR_{7.5}$ and CRR has been described by Been and Jefferies [29] as:

$$\boldsymbol{CRR = CRR_{7.5}K_MK_\sigma K_\alpha} \qquad (6)$$

The corrected CRR is compared with the corresponding CSR to obtain the FoS (factor of safety) as shown in Figures 17 and 18. Both the SGH and PS15 plots mirrored the state parameters of the soil at the respective depths. The liquefaction evaluation was such that the SGH site would liquefy at the layers above 6m for the M6.2 Christchurch earthquake. The soil layers below 7m would not liquefy due to the fact that the soil is predominantly composed of fines which have PI values of approximately 7.7, a bit greater than the threshold PI value of 7 considered to be exempt from liquefaction [34].

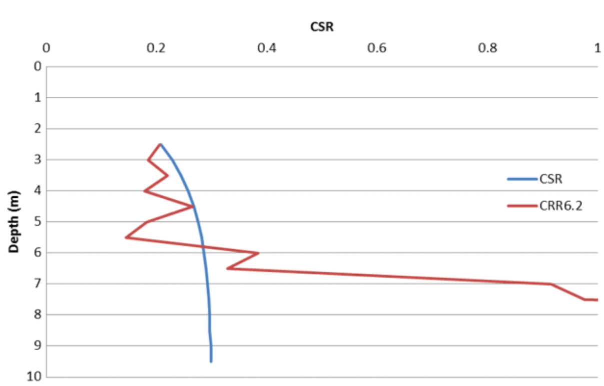

Figure 17: Comparison between CSR and CRR at St George's Hospital site.

The result of comparison for PS15 site is illustrated in Figure 18, where it can be seen that the FoS is much lower and the liquefiable layer thicker, extending from 1m to 9.5m.

These results are consistent with the visual manifestation observed at the sites, which was minor liquefaction at the SGH site and moderate liquefaction at the PS15 site.

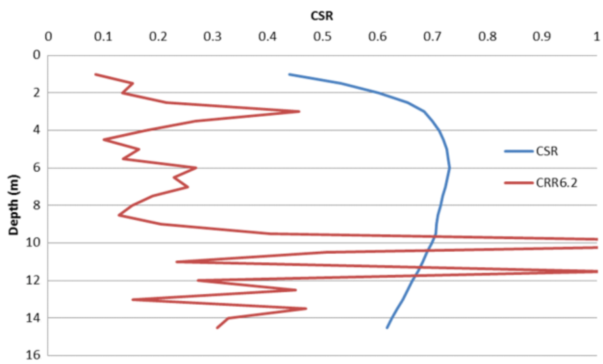

Figure 18: Comparison between CSR and CRR at Pumpstation 15 site.

A correlation between the visual observation on the sites and the liquefaction evaluation results was sought to support this method. As the soil profiles did not extend to 20m deep due to the limitations of the sDMT test, the LPI was unusable in this scenario. Instead, the LSN was used instead. The formulation of the LSN was explained earlier and LSN values were calculated for the SGH and PS15 sites.

The SGH site had a LSN value of 13 considering only the layers between 2.5-9.5m, and it was determined that the surface manifestation contribution of any liquefaction occurring below this level was negligible (due to the nature of the depth weighting function). The above value corresponds to minor surface manifestation with some sand boils, which was consistent with the site observation.

The PS15 site recorded a much higher LSN value of 32 for the layer from 1-14.5m, with the effect of any liquefaction occurring below this level deemed to be negligible in terms of surface manifestation due to the large distance from the surface. The much larger LSN value was a result of many factors including a higher ground water table, thinner overlying non-liquefiable crust, and a larger overall PGA. The LSN value of 32.4 is agreeable with visual observations, i.e. moderate amount of liquefaction and sand boils at the site. The LSN factor reinforced the applicability of this method as the predicted outcome related positively with the in-situ observation.

Comparison with Empirical Methods

The proposed critical state soil mechanics (CSSM)-based method is next compared with other existing empirical methods: i.e. methods by Robertson and Cabal [39], Moss et al. [17], Boulanger and Idriss [5] and Kayen et al. [4], and these are referred herein as R&C12, MEA06, B&I14 and KEA13 respectively. These methods were not used to validate the proposed method based on CSSM, but rather for comparison purposes, as it is known that different empirical methods occasionally yield contradictory predictions [40, 41]. These empirical methods are also compared in terms of visual manifestations observed at both SGH and PS15 sites. The CPT-based methods are tested for the entire 20m depth as the CPT data was available for the whole 20m depth, while KEA13 is tested for the entire depth of the sDMT, which was 11.5m for the SGH site and 14.5 for the PS15 site.

The calculations were again performed using the parameters used above, i.e., same PGA and same ground water levels. The probability of liquefaction (P_L) corresponding to 50% was used as the deterministic boundary for the probabilistic-based methods (MEA06 and KEA2013). The soil unit weights for R&C12 were estimated via CPT data provided within the method. For KEA13, the shear wave velocity-based method, the soil unit weight was obtained from empirical correlation of the dilatometer readings as part of the sDMT testing process. For the other methods, soil unit weights of 17kN/m^3 and 19.5kN/m^3 were assumed for layers above and below the water table, respectively.

St George's Hospital

The empirical methods were used to determine the liquefaction potential of the soil profile at the SGH site, and the results are shown in Figure 19. R&C12 and B&I14 seemed to unanimously agree that the majority of the entire layer from 2.5m to 20m would liquefy. In contrast, MEA06's and KEA13's prediction is that only intermittent liquefaction would occur. With regards to KEA13, even if liquefaction were to occur below 11.5m, the overlying non-liquefiable layer would be too thick to produce any surface manifestation. Therefore, it can be said that all four methods yield somewhat different estimates.

A comparison of these liquefaction predictions with visual observations through the LPI parameter is shown in Figure 21(a). LPI values of 24, 17 and 27 were obtained for R&C12, MEA06 and B&I14, respectively. However, all three of these LPI values are deemed too high compared with the minor liquefaction observed. Note that LPI values in excess of 15 are supposed to be related to very severe liquefaction manifestation [18, 19].

This may be an artefact of the uniqueness of Christchurch soils from existing trends. Another possible factor would be the thick non-liquefiable crust overlying the liquefiable layer which would suppress any sign of surface manifestation [40]. Ishihara [42] indicated that it would take a 3m thick non-liquefiable layer to suppress the effects of surface manifestation, regardless of the thickness of the underlying liquefiable layer.

The LPI parameter was unobtainable for the KEA13 method due to the insufficient depth considered, but it can be deduced that the LPI parameter would be very small. This was emphasised by the LSN parameter which was calculated to be around 8, which means that almost no liquefaction is predicted to occur. However, this prediction by KEA13 is also not in agreement with the minor liquefaction observed at the SGH site. Thus, it appears that, based on these site results, there is no correlation between visual observations and the empirical method predictions, unlike the CSSM-based method.

Pumpstation 15

Similar type of comparison was made on the soil profile at PS15 site, and the results are presented in Figure 20. All three CPT-based methods show similar trend, i.e. the CRR would increase or decrease in all of the methods simultaneously. Similar closeness in trend is also observed in the SGH site.

KEA13 predicted that the soil layer from 1-9.5m would liquefy while the layers below this would not, which coincide with the increase in liquefaction resistance at 9-10m predicted by the CPT-based methods, but is otherwise different. When compared with SGH site, the PS15 soil profile can be seen to have a much larger CRR overall and is more resistant to liquefaction. This is also shown by the presence of thick non-liquefiable layers in the soil profile, which was absent in the SGH soil profile. However, the PGA, and thus the CSR, experienced at the PS15 site was much larger than that at the SGH site due to the proximity to the epicentre, the main reason for the soil profile to have liquefied.

Calculating the LPI values, it was found that similar to SGH, B&I14 predicted the highest LPI while MEA06 yielded the lowest LPI as indicated in Figure 21(b). These values are surprisingly smaller than that of SGH, although PS15 site was observed to have a more severe expression of liquefaction at the surface.

The V_s-based method KEA13 resulted in LSN value of around 29, which corresponded well with that obtained from the CSSM- based method. It was also noted that both the CSSM-based method and the KEA13 method yielded very similar shape and trend for the CRR for this PS15 site, something not observed in the SGH site. These LSN values indicate a moderate expression of liquefaction on the surface, which agree with site observations.

Overall, the empirical methods R&C12, B&I14, MEA06 and KEA13 are shown to be inaccurate and inconsistent in their predictions of liquefaction susceptibility for the sites considered in this study. The exception was KEA13's reasonable prediction for the PS15 site, and the similar CRR trend to that observed with the CSSM-based method. The CPT based methods were found to consistently over-predict the severity of liquefaction compared to actual occurrence, while KEA13 predicted no liquefaction at all for the SGH site. This highlights the importance of the development of the CSSM-based method for use in liquefaction evaluation for these two sites and, potentially, for the rest of Christchurch.

CONCLUDING REMARKS

This study proposed a method to evaluate the onset of liquefaction in the Christchurch region using the critical state soil mechanics-based approach. The study was carried considering the notion that current empirical methods would not work on Christchurch's peculiar soils. Moreover, it was desired to explain the onset of liquefaction phenomenon using more scientific-based approach.

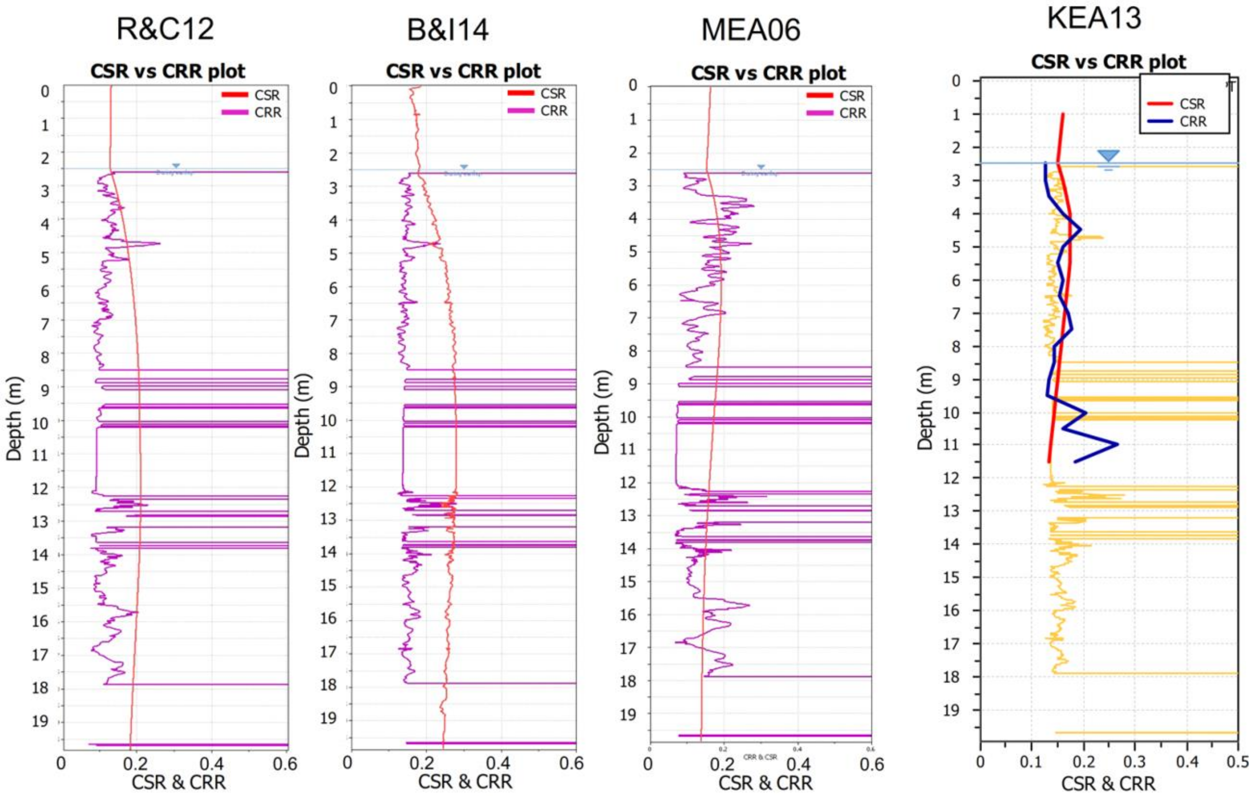

Figure 19: Liquefaction evaluation plots for SGH site using: (a) Robertson and Cabal (2012 [39]); (b) Boulanger and Idriss (2014) [5]; (c) Moss et al. (2006) [17]; and (d) Kayen et al. (2013 [4]).

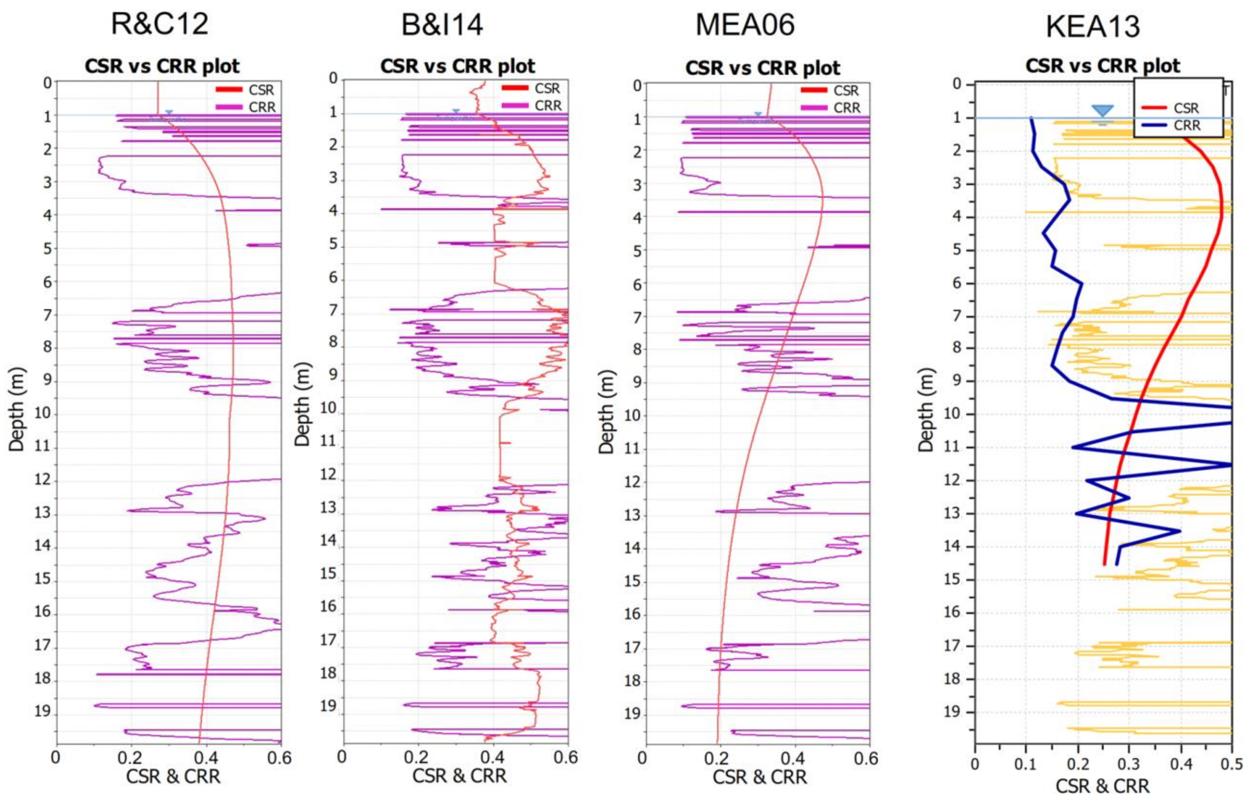

Figure 20: Liquefaction evaluation plots for PS15 site using: (a) Robertson and Cabal (2012) [39]; (b) Boulanger and Idriss (2014) [5]; (c) Moss et al. (2006) [17]; and (d) Kayen et al. (2013) [4].

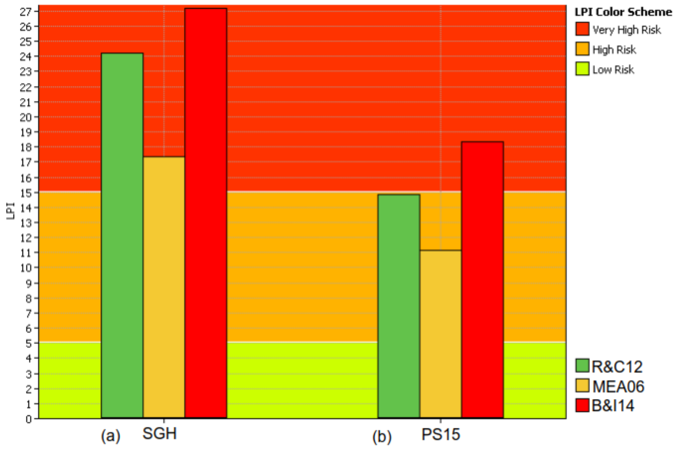

Figure 21: Comparison of LPI values at: (a) SGH site; and (b) PS15 site.

The proposed critical state soil mechanics-based simplified procedure incorporated a strong theoretical basis, with each step explained scientifically using critical state soil mechanics.

A simple step by step summary of how this procedure is executed is as follows:

- The critical state line(s) of the critical soil layer(s) needs to be ascertained first. The Γ is approximated to be equivalent to the e_{max} of the soil, while the λ is approximated using a relationship with the void ratio range of the soil.
- The in-situ void ratio is ascertained via shear wave velocity readings, possibly through sDMT and sCPT. The shear wave velocity is directly related to the void ratio using Hardin and Richart's [8] relationships.
- The state parameter, ψ, is then obtainable and a relationship obtained for Christchurch soils is used to estimate the corresponding CRR, which should be adjusted for overburden pressure and earthquake magnitude, and ultimately compared with the CSR to calculate the factor of safety.
- The LSN can be used to obtain a representation of how severe the liquefaction is going to manifest on the surface.

The proposed method has been shown to work well for the two sites investigated in Christchurch, namely at St George's Hospital and Pumpstation 15. Predictions from the proposed model matched very well with the observations at the site.

This simplified procedure, however, has been limited to Christchurch soils specifically, as it was shown that the soils in the city is exclusive to the region and do not fit into existing soil database trends. This means that this procedure will not be affected by other soils, and will be more accurate for the Christchurch specific soils. Further evaluation of other case histories with different soil profiles in Christchurch would aid in refining this procedure and confirm the liquefaction predictions and earthquake observations.

The use of the existing empirical methods to determine the onset of liquefaction in Christchurch was shown to be unsuccessful based on the limited sites considered in the study. The CPT-based methods were shown to vastly overestimate the amount of liquefaction and soil damage, while the V_s-based method was shown to be wanting at the SGH site which was characterised by very high fines content throughout the critical layer. Other studies performed in Christchurch site [40] yielded generally the same conclusion that the empirical methods did not yield a response appropriate to what actually occurred at the sites.

Note that, as mentioned earlier, only two sites in Christchurch were examined to show that the CSSM-based method is robust and better at predicting liquefaction triggering compared to the CPT-based methods. It is recommended that more case history sites with different soil profiles should be examined in the future to see if there is as good an agreement between the liquefaction predictions and the earthquake observations at other locations. Moreover, it has been mentioned that some sites in Christchurch may be partially saturated to deeper locations even if the water table is considerably higher; therefore, how to take into account partial saturation in any CSSM-, CPT- and V_S-based liquefaction triggering assessment methods should be considered in future research.

ACKNOWLEDGMENT

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REFERENCES

1. Orense RP, Kiyota T, Yamada S, Cubrinovski M., Hosono Y, Okamura M and Yasuda S (2011). "Comparison of liquefaction features observed during the 2010 and 2011 Canterbury earthquakes". *Seismological Research Letters*, 82(6): 905-918.
2. Green RA, Wood C, Cox B, Cubrinovski M, Wotherspoon L, Bradley B and Rix G (2011). "Use of DCP and SASW tests to evaluate liquefaction potential: Predictions vs. observations during the recent New Zealand earthquakes". *Seismological Research Letters*, 82(6): 927-938.
3. Robertson PK and Wride CE (1998). "Evaluating cyclic liquefaction potential using the cone penetration test". *Canadian Geotechnical Journal*, 35(3): 442-459.
4. Kayen R, Moss RES, Thompson EM, Seed RB, Cetin KO, Der Kiureghian A and Tokimatsu K (2013). "Shear-wave velocity-based probabilistic and deterministic assessment of seismic soil liquefaction potential". *Journal of Geotechnical and Geoenvironmental Engineering*, 139(3): 407-419.
5. Boulanger R and Idriss I (2014). "*CPT and SPT Based Liquefaction Triggering Procedures*". Report No. UCD/CGM-14/01, University of California, Davis, CA: Center for Geotechnical Modeling, Department of Civil and Environmental Engineering.
6. Stringer M, Beyzaei C, Cubrinovski M, Bray J, Riemer M, Jacka M and Wentz F (2015). "Liquefaction

characteristics of Christchurch silty soils: Gainsborough Reserve". *6th International Conference on Earthquake Geotechnical Engineering*, Christchurch, NZ.

7. Schofield A and Wroth P (1968). "*Critical State Soil Mechanics*". (Illustrated Ed.). London: McGraw-Hill, 1968.
8. Hardin B and Richart Jr F (1963). "Elastic wave velocities in granular soils". *Journal of Soil Mechanics & Foundations Division*, **89**(1): 33-66.
9. Herle I and Gudehus G (1999). "Determination of parameters of a hypoplastic constitutive model from properties of grain assemblies". *Mechanics of Cohesive-frictional Materials*, 4(5): 461-486.
10. Cubrinovski M and Ishihara K (2000). "Flow potential of sandy soils with different grain compositions". *Soils and Foundations*, 40(4): 103-119.
11. Brown L, Beetham R, Paterson B and Weeber J (1995). Geology of Christchurch, New Zealand". *Environmental & Engineering Geoscience,* 1(4): 427-488.
12. GNS Science (2014). "*M 6.3, Christchurch, 22 February 2011*". Retrieved from http://info.geonet.org.nz/display/quake/M+6.3,+Christchurch,+22+February+2011
13. Been K and Jefferies MG (1985). "A state parameter for sands". *Geotechnique,* 35(2): 99-112.
14. Seed H, Lee K and Idriss I (1969). "Analysis of Sheffield dam failure". *Journal of Soil Mechanics & Foundations Division*, 95(6): 1453-1490.
15. Seed HB and Idriss IM (1971). "Simplified procedure for evaluating soil liquefaction potential". *Journal of the Soil Mechanics and Foundations Division*, 97(9): 1249-1273.
16. Andrus RD and Stokoe II KH (2000). "Liquefaction resistance of soils from shear-wave velocity". *Journal of Geotechnical and Geoenvironmental Engineering, ASCE*, 126(11): 1015-1025.
17. Moss RE, Seed RB, Kayen RE, Stewart JP, Der Kiureghian A and Cetin KO (2006). "CPT-based probabilistic and deterministic assessment of in situ seismic soil liquefaction potential". *Journal of Geotechnical and Geoenvironmental Engineering*, 132(8): 1032-1051.
18. Iwasaki T, Tatsuoka F, Tokida K and Yasuda S (1978). "A practical method for assessing soil liquefaction potential based on case studies at various sites in Japan". *Proceedings 2nd International Conference on Microzonation*, San Francisco, CA, 885-896.
19. Toprak S and Holzer TL (2003). "Liquefaction potential index: Field assessment". *Journal of Geotechnical and Geoenvironmental Engineering*, 129(4): 315-322.
20. Li DK, Juang CH and Andrus RD (2006). "Liquefaction potential index: A critical assessment using probability concept". *Taiwan Geotechnical Society Journal of Geotechnical Engineering*, 1(1): 11-24.
21. van Ballegooy S and Malan P (2013). "*Liquefaction Vulnerability Study*". Report to Earthquake Commission, Tonkin and Taylor, Christchurch, 59pp, http://www.eqc.govt.nz/sites/public_files/documents/liquefaction-vulnerability-study-final.pdf.
22. Robertson P, Sasitharan S, Cunning J and Sego D (1995). "Shear-wave velocity to evaluate in-situ state of Ottawa sand". *Journal of Geotechnical Engineering*, 121(3): 262-273.
23. Robertson P and Fear C (1995). "Application of CPT to evaluate liquefaction potential". *Proceedings, International Symposium on CPT*, Linkoping, Sweden, Swedish Geotechnical Society, Vol. 3, pp. 57-79.
24. Chang HN (2005). "The Relationship between Void Ratio and Shear Wave Velocity of Gold Tailings". ME Thesis, University of Pretoria, South Africa.
25. Wood DM (1990). "*Soil Behaviour and Critical State Soil Mechanics*". Cambridge University Press, Cambridge, UK, 112-136.
26. Niemunis A (2003). "*Extended Hypoplastic Models for Soils*". Inst. für Grundbau und Bodenmechanik, http://www.pg.gda.pl/~aniem/pap-zips/habb-30-7-2003.pdf.
27. Niemunis A and Herle I (1997). "Hypoplastic model for cohesionless soils with elastic strain range". *Mechanics of Cohesive-Frictional Materials,* 2(4): 279-299.
28. Canterbury Geotechnical Database (2015). "*Liquefaction and Lateral Spreading Observations*". Retrieved from https://canterburygeotechnicaldatabase.projectorbit.com/
29. Been K and Jefferies M (2006). "*Soil Liquefaction: A Critical State Approach*". Abingdon, Oxon: Taylor & Francis.
30. Standards New Zealand (1986). NZS 4402:1986 - Methods of testing soils for civil engineering purposes".
31. ASTM D4767 (2004). "Standard Test Method for Consolidated Undrained Triaxial Compression Test for Cohesive Soils". Annual Book of ASTM Standards, 913-925.
32. Rees SD (2010). "Effects of Fines on the Undrained Behaviour of Christchurch Sandy Soils". PhD Thesis, University of Canterbury.
33. Thevanayagam S and Martin G (2002). "Liquefaction in silty soils—screening and remediation issues". *Soil Dynamics and Earthquake Engineering*, 22(9): 1035-1042.
34. Boulanger RW and Idriss IM (2006). "Liquefaction susceptibility criteria for silts and clays". *Journal of Geotechnical and Geoenvironmental Engineering, ASCE*, 132(11): 1413-1426.
35. Cubrinovski M and Ishihara K (2002). "Maximum and minimum void ratio characteristics of sands". *Soils and Foundations*, 42(6): 65-78.
36. Mašín D (2005). "A hypoplastic constitutive model for clays". *International Journal for Numerical and Analytical Methods in Geomechanics*, 29(4): 311-336.
37. Bouckovalas GD, Andrianopoulos KI and Papadimitriou AG (2003). "A critical state interpretation for the cyclic liquefaction resistance of silty sands". *Soil Dynamics and Earthquake Engineering*, 23(2): 115-125.
38. Hong Y, Yang Z, Orense RP and Lu Y (2015). "Investigation of sand-tire mixtures as liquefaction remedial measure". *Tenth Pacific Conference on Earthquake Engineering*, Sydney, Australia, 8pp.
39. Robertson P and Cabal K (2012). "*Guide to Cone Penetration Testing for Geotechnical Engineering*". Gregg Drilling & Testing. Inc., 5th Edition, California.
40. Wotherspoon L, Orense R, Green R, Bradley B, Cox B and Wood C (2014). "Analysis of liquefaction characteristics at Christchurch strong motion stations". *New Zealand - Japan Workshop on Soil Liquefaction during Recent Large-Scale Earthquakes*, (ISBN 9781138026438), pp. 33-45.
41. Green RA, Cubrinovski M, Cox B, Wood C, Wotherspoon L, Bradley B and Maurer B (2014). "Select liquefaction case histories from the 2010-2011 Canterbury earthquake sequence". *Earthquake Spectra*, 30(1): 131-153.
42. Ishihara K (1985). "Stability of natural deposits during earthquakes". *11th International Conference on Soil Mechanics and Foundation Engineering*, Rotterdam, The Netherlands, Vol. 1, pp. 321-376.